CONDUCTION HEAT TRANSFER

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The ability to analyze a problem involves a combination of inherent insight and experience. *The former, unfortunately, cannot be learned, but depends on the individual. However, the latter is of equal importance, and can be gained with* patient study.

2–9. Methods of Formulation

In the preceding sections of this chapter we have established the general formulation of conduction phenomena. We might now expect to obtain the formulation of any specific problem from the general formulation. This, of course, is possible, but it is not always convenient, especially if the problem under consideration is to be lumped in one or more directions. (This point will be clarified by the problem of Fig. 2–27.) Furthermore, the application of the general formulation to a specific problem is a mathematical process which eliminates the physics of the formulation, an important aspect of practical problems. By contrast, the physical approach which will be stressed in this text treats each problem individually from the start of its formulation by bringing the physics into each phase of the formulation. To illustrate this statement let us compare the two methods in the light of three problems requiring the one-dimensional formulation of the first law of thermodynamics.

The first problem is that of the one-dimensional cartesian system shown in Fig. 2–25. When we equate the time rate of change of internal energy to the net heat transfer across the boundaries of the system, the physical approach yields

$$\rho \frac{\partial u}{\partial t} + \frac{\partial q_x}{\partial x} = 0. \qquad (2\text{–}117)$$

The general formulation, reduced to the one-dimensional cartesian form of Eq. (2–57), gives the same result.

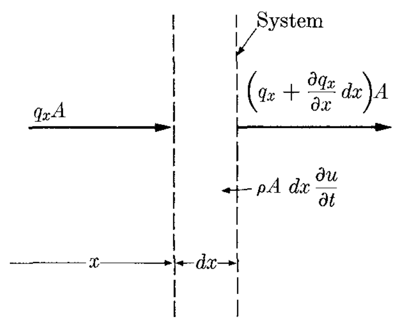

FIG. 2–25

Next, let us consider an insulated solid rod of radius R, cross section A, and periphery P (Fig. 2–26). By either method, the first law of thermodynamics stated for the one-dimensional system shown in Fig. 2–26 yields the result of the previous problem, Eq. (2–117).

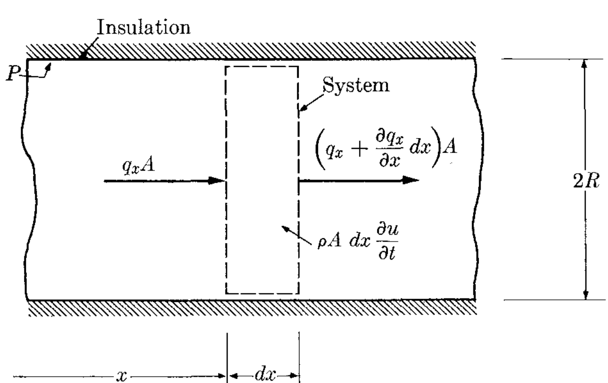

FIG. 2–26

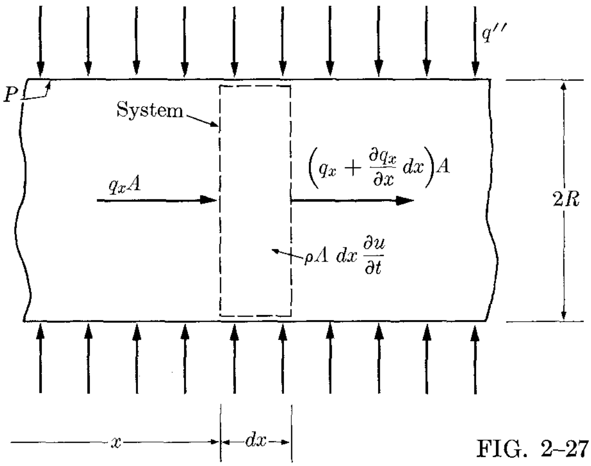

FIG. 2-27

Finally, consider the solid rod of the previous problem now subjected to the uniform peripheral heat flux q'' (Fig. 2-27). The physical approach, in which we apply the first law to the one-dimensional system shown in Fig. 2-27, results in

$$\rho\frac{\partial u}{\partial t} + \frac{\partial q_x}{\partial x} = \frac{q''P}{A}. \tag{2-118}$$

By contrast, the one-dimensional form of the general formulation, leading again to Eq. (2-117), does not include the effect of the peripheral heat flux. This difficulty, however, may be circumvented by considering instead the two-dimensional form of Eq. (2-57),

$$\rho\frac{\partial u^*}{\partial t} + \frac{\partial q_x^*}{\partial x} + \frac{1}{r}\frac{\partial}{\partial r}(rq_r^*) = 0, \tag{2-119}$$

where u^*, q_x^*, and q_r^* now depend on r as well as x and t. Next, averaging Eq. (2-119) radially, that is, multiplying each term by $2\pi r\,dr$ and integrating the result over the interval $(0, R)$, yields

$$A\rho\frac{\partial u}{\partial t} + A\frac{\partial q_x}{\partial x} + 2\pi rq_r\Big|_0^R = 0, \tag{2-120}$$

which is identical to Eq. (2-118). Here the radially averaged value of a dependent variable, say u, is defined as

$$u(x, t) = \frac{2\pi}{A}\int_0^R ru^*(r, x, t)\,dr.$$

The discussion on the foregoing three examples may be generalized as follows. A given problem may be formulated either by considering the appropriate specific case of the general formulation or by following, from the start, an individual formulation suitable to the problem. Whenever the general formulation is available the former method may be used, but this requires the mathematical interpretation of the general formulation in the light of the problem under consideration. The latter method, on the other hand, involves following certain steps in a basic procedure for individual formulation, given below. For one- or multidimensional problems which are formulated to include all dimensions of the problem (such as the first two of the foregoing examples), the general or mathematical approach proves to be slightly shorter than the individual or physical approach. However, for multidimensional problems which we wish to formulate in fewer dimensions, that is, which we wish to lump in one or more directions (such as the third example), the mathematical approach, requiring an averaging process, becomes lengthy and inconvenient.

The foregoing argument and the emphasis, in this text, on practical applications of the study of heat transfer thus suggest that the physical approach be the preferred method of formulation. For convenience and later reference, this method of formulation is summarized in the following five steps:

(i) *Define an appropriate system or control volume.* This step includes the selection of (a) a coordinate system, (b) a lumped or distributed formulation, and (c) a system or control volume in terms of (a) and (b).

(ii) *State the general laws for* (i). The general laws, except in their lumped forms, are written in terms of a coordinate system. The differential forms of these laws depend on the direction but not the origin of coordinates, whereas the integral forms depend on the origin as well as the direction of coordinates. Although the differential forms apply locally, the lumped and integral forms are stated for the entire system or control volume.

(iii) *State the particular laws for* (ii). The particular law describing the diffusion of heat (or momentum, mass, or electricity) is differential, applies locally, and depends on the direction but not the origin of coordinates.

(iv) *Obtain the governing equation from* (ii) *and* (iii). This, such as the equation of conduction, may be an algebraic, differential, or other equation involving the desired dependent variable, say the temperature, as the only unknown. The governing equation (except for its flow terms) is independent of the origin and direction of coordinates.

(v) *Specify the initial and/or boundary conditions pertinent to* (iv). These conditions depend on the origin as well as the direction of coordinates.

2–10. Examples

In this section the emphasis is placed on formulation; however, for those problems whose formulation leads to an ordinary differential equation of first order or to one of second order with constant coefficients we shall also give the solution.