Chapter 1

Introduction

1.1 Preliminary Remarks

Fluid mechanics is the study of fluids either in motion (fluid *dynamics*) or at rest (fluid *statics*) and the subsequent effects of the fluid upon the boundaries, which may be either solid surfaces or interfaces with other fluids. Both gases and liquids are classified as fluids, and the number of fluids engineering applications is enormous: breathing, blood flow, swimming, pumps, fans, turbines, airplanes, ships, rivers, windmills, pipes, missiles, icebergs, engines, filters, jets, and sprinklers, to name a few. When you think about it, almost everything on this planet either is a fluid or moves within or near a fluid.

The essence of the subject of fluid flow is a judicious compromise between theory and experiment. Since fluid flow is a branch of mechanics, it satisfies a set of well-documented basic laws, and thus a great deal of theoretical treatment is available. However, the theory is often frustrating, because it applies mainly to idealized situations which may be invalid in practical problems. The two chief obstacles to a workable theory are geometry and viscosity. The basic equations of fluid motion (Chap. 4) are too difficult to enable the analyst to attack arbitrary geometric configurations. Thus most textbooks concentrate on flat plates, circular pipes, and other easy geometries. It is possible to apply numerical computer techniques to complex geometries, and specialized textbooks are now available to explain the new *computational fluid dynamics* (CFD) approximations and methods [1, 2, 29].[1] This book will present many theoretical results while keeping their limitations in mind.

The second obstacle to a workable theory is the action of viscosity, which can be neglected only in certain idealized flows (Chap. 8). First, viscosity increases the difficulty of the basic equations, although the boundary-layer approximation found by Ludwig Prandtl in 1904 (Chap. 7) has greatly simplified viscous-flow analyses. Second, viscosity has a destabilizing effect on all fluids, giving rise, at frustratingly small velocities, to a disorderly, random phenomenon called *turbulence*. The theory of turbulent flow is crude and heavily backed up by experiment (Chap. 6), yet it can be quite serviceable as an engineering estimate. Textbooks now present digital-computer techniques for turbulent-flow analysis [32], but they are based strictly upon empirical assumptions regarding the time mean of the turbulent stress field.

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[1] Numbered references appear at the end of each chapter.

Thus there is theory available for fluid-flow problems, but in all cases it should be backed up by experiment. Often the experimental data provide the main source of information about specific flows, such as the drag and lift of immersed bodies (Chap. 7). Fortunately, fluid mechanics is a highly visual subject, with good instrumentation [4, 5, 35], and the use of dimensional analysis and modeling concepts (Chap. 5) is widespread. Thus experimentation provides a natural and easy complement to the theory. You should keep in mind that theory and experiment should go hand in hand in all studies of fluid mechanics.

1.2 The Concept of a Fluid

From the point of view of fluid mechanics, all matter consists of only two states, fluid and solid. The difference between the two is perfectly obvious to the layperson, and it is an interesting exercise to ask a layperson to put this difference into words. The technical distinction lies with the reaction of the two to an applied shear or tangential stress. *A solid can resist a shear stress by a static deformation; a fluid cannot*. Any shear stress applied to a fluid, no matter how small, will result in motion of that fluid. The fluid moves and deforms continuously as long as the shear stress is applied. As a corollary, we can say that a fluid at rest must be in a state of zero shear stress, a state often called the hydrostatic stress condition in structural analysis. In this condition, Mohr's circle for stress reduces to a point, and there is no shear stress on any plane cut through the element under stress.

Given the definition of a fluid above, every layperson also knows that there are two classes of fluids, *liquids* and *gases*. Again the distinction is a technical one concerning the effect of cohesive forces. A liquid, being composed of relatively close-packed molecules with strong cohesive forces, tends to retain its volume and will form a free surface in a gravitational field if unconfined from above. Free-surface flows are dominated by gravitational effects and are studied in Chaps. 5 and 10. Since gas molecules are widely spaced with negligible cohesive forces, a gas is free to expand until it encounters confining walls. A gas has no definite volume, and when left to itself without confinement, a gas forms an atmosphere which is essentially hydrostatic. The hydrostatic behavior of liquids and gases is taken up in Chap. 2. Gases cannot form a free surface, and thus gas flows are rarely concerned with gravitational effects other than buoyancy.

Figure 1.1 illustrates a solid block resting on a rigid plane and stressed by its own weight. The solid sags into a static deflection, shown as a highly exaggerated dashed line, resisting shear without flow. A free-body diagram of element A on the side of the block shows that there is shear in the block along a plane cut at an angle θ through A. Since the block sides are unsupported, element A has zero stress on the left and right sides and compression stress $\sigma = -p$ on the top and bottom. Mohr's circle does not reduce to a point, and there is nonzero shear stress in the block.

By contrast, the liquid and gas at rest in Fig. 1.1 require the supporting walls in order to eliminate shear stress. The walls exert a compression stress of $-p$ and reduce Mohr's circle to a point with zero shear everywhere, i.e., the hydrostatic condition. The liquid retains its volume and forms a free surface in the container. If the walls are removed, shear develops in the liquid and a big splash results. If the container is tilted, shear again develops, waves form, and the free surface seeks a horizontal configura-

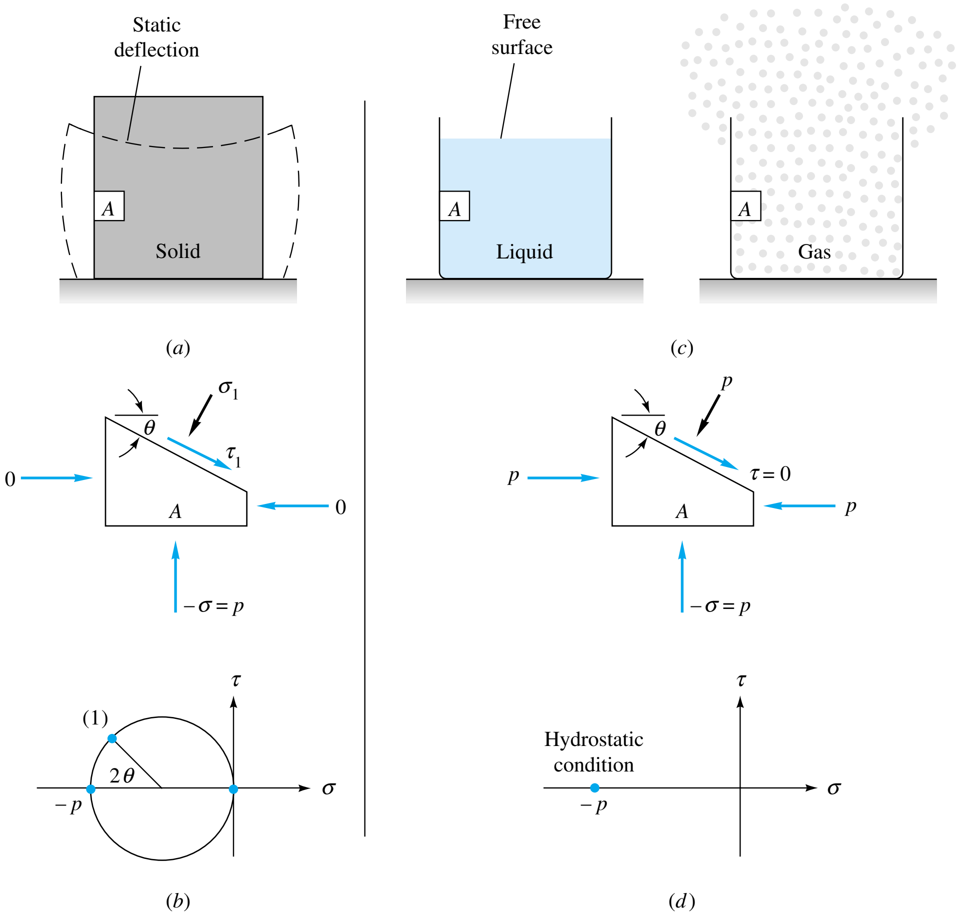


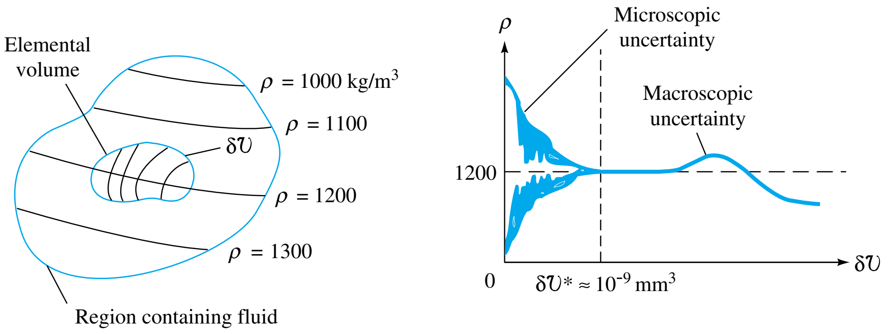
Fig. 1.1 A solid at rest can resist shear. (*a*) Static deflection of the solid; (*b*) equilibrium and Mohr's circle for solid element *A*. A fluid cannot resist shear. (*c*) Containing walls are needed; (*d*) equilibrium and Mohr's circle for fluid element *A*.

tion, pouring out over the lip if necessary. Meanwhile, the gas is unrestrained and expands out of the container, filling all available space. Element *A* in the gas is also hydrostatic and exerts a compression stress $-p$ on the walls.

In the above discussion, clear decisions could be made about solids, liquids, and gases. Most engineering fluid-mechanics problems deal with these clear cases, i.e., the common liquids, such as water, oil, mercury, gasoline, and alcohol, and the common gases, such as air, helium, hydrogen, and steam, in their common temperature and pressure ranges. There are many borderline cases, however, of which you should be aware. Some apparently "solid" substances such as asphalt and lead resist shear stress for short periods but actually deform slowly and exhibit definite fluid behavior over long periods. Other substances, notably colloid and slurry mixtures, resist small shear stresses but "yield" at large stress and begin to flow as fluids do. Specialized textbooks are devoted to this study of more general deformation and flow, a field called *rheology* [6]. Also, liquids and gases can coexist in two-phase mixtures, such as steam-water mixtures or water with entrapped air bubbles. Specialized textbooks present the analysis of such *two-phase flows* [7]. Finally, there are situations where the distinction between a liquid and a gas blurs. This is the case at temperatures and pressures above the so-called *critical point* of a substance, where only a single phase exists, primarily resembling a gas. As pressure increases far above the critical point, the gaslike substance becomes so dense that there is some resemblance to a liquid and the usual thermodynamic approximations like the perfect-gas law become inaccurate. The critical temperature and pressure of water are $T_c = 647$ K and $p_c = 219$ atm,[2] so that typical problems involving water and steam are below the critical point. Air, being a mixture of gases, has no distinct critical point, but its principal component, nitrogen, has $T_c = 126$ K and $p_c = 34$ atm. Thus typical problems involving air are in the range of high temperature and low pressure where air is distinctly and definitely a gas. This text will be concerned solely with clearly identifiable liquids and gases, and the borderline cases discussed above will be beyond our scope.

1.3 The Fluid as a Continuum

We have already used technical terms such as *fluid pressure* and *density* without a rigorous discussion of their definition. As far as we know, fluids are aggregations of molecules, widely spaced for a gas, closely spaced for a liquid. The distance between molecules is very large compared with the molecular diameter. The molecules are not fixed in a lattice but move about freely relative to each other. Thus fluid density, or mass per unit volume, has no precise meaning because the number of molecules occupying a given volume continually changes. This effect becomes unimportant if the unit volume is large compared with, say, the cube of the molecular spacing, when the number of molecules within the volume will remain nearly constant in spite of the enormous interchange of particles across the boundaries. If, however, the chosen unit volume is too large, there could be a noticeable variation in the bulk aggregation of the particles. This situation is illustrated in Fig. 1.2, where the "density" as calculated from molecular mass δm within a given volume $\delta\mathcal{V}$ is plotted versus the size of the unit volume. There is a limiting volume $\delta\mathcal{V}^*$ below which molecular variations may be important and



(*a*) (*b*)

Fig. 1.2 The limit definition of continuum fluid density: (*a*) an elemental volume in a fluid region of variable continuum density; (*b*) calculated density versus size of the elemental volume.

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[2] One atmosphere equals 2116 lbf/ft^2 = 101,300 Pa.

above which aggregate variations may be important. The *density* ρ of a fluid is best defined as

$$\rho = \lim_{\delta \mathcal{V} \to \delta \mathcal{V}^*} \frac{\delta m}{\delta \mathcal{V}} \tag{1.1}$$

The limiting volume $\delta\mathcal{V}^*$ is about 10^{-9} mm^3 for all liquids and for gases at atmospheric pressure. For example, 10^{-9} mm^3 of air at standard conditions contains approximately 3×10^7 molecules, which is sufficient to define a nearly constant density according to Eq. (1.1). Most engineering problems are concerned with physical dimensions much larger than this limiting volume, so that density is essentially a point function and fluid properties can be thought of as varying continually in space, as sketched in Fig. 1.2*a*. Such a fluid is called a *continuum,* which simply means that its variation in properties is so smooth that the differential calculus can be used to analyze the substance. We shall assume that continuum calculus is valid for all the analyses in this book. Again there are borderline cases for gases at such low pressures that molecular spacing and mean free path[3] are comparable to, or larger than, the physical size of the system. This requires that the continuum approximation be dropped in favor of a molecular theory of rarefied-gas flow [8]. In principle, all fluid-mechanics problems can be attacked from the molecular viewpoint, but no such attempt will be made here. Note that the use of continuum calculus does not preclude the possibility of discontinuous jumps in fluid properties across a free surface or fluid interface or across a shock wave in a compressible fluid (Chap. 9). Our calculus in Chap. 4 must be flexible enough to handle discontinuous boundary conditions.

1.4 Dimensions and Units

A *dimension* is the measure by which a physical variable is expressed quantitatively. A *unit* is a particular way of attaching a number to the quantitative dimension. Thus length is a dimension associated with such variables as distance, displacement, width, deflection, and height, while centimeters and inches are both numerical units for expressing length. Dimension is a powerful concept about which a splendid tool called *dimensional analysis* has been developed (Chap. 5), while units are the nitty-gritty, the number which the customer wants as the final answer.

Systems of units have always varied widely from country to country, even after international agreements have been reached. Engineers need numbers and therefore unit systems, and the numbers must be accurate because the safety of the public is at stake. You cannot design and build a piping system whose diameter is D and whose length is L. And U.S. engineers have persisted too long in clinging to British systems of units. There is too much margin for error in most British systems, and many an engineering student has flunked a test because of a missing or improper conversion factor of 12 or 144 or 32.2 or 60 or 1.8. Practicing engineers can make the same errors. The writer is aware from personal experience of a serious preliminary error in the design of an aircraft due to a missing factor of 32.2 to convert pounds of mass to slugs.

In 1872 an international meeting in France proposed a treaty called the Metric Convention, which was signed in 1875 by 17 countries including the United States. It was an improvement over British systems because its use of base 10 is the foundation of our number system, learned from childhood by all. Problems still remained because

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[3] The mean distance traveled by molecules between collisions.

even the metric countries differed in their use of kiloponds instead of dynes or newtons, kilograms instead of grams, or calories instead of joules. To standardize the metric system, a General Conference of Weights and Measures attended in 1960 by 40 countries proposed the *International System of Units* (SI). We are now undergoing a painful period of transition to SI, an adjustment which may take many more years to complete. The professional societies have led the way. Since July 1, 1974, SI units have been required by all papers published by the American Society of Mechanical Engineers, which prepared a useful booklet explaining the SI [9]. The present text will use SI units together with British gravitational (BG) units.

Primary Dimensions

In fluid mechanics there are only four *primary dimensions* from which all other dimensions can be derived: mass, length, time, and temperature.[4] These dimensions and their units in both systems are given in Table 1.1. Note that the kelvin unit uses no degree symbol. The braces around a symbol like $\{M\}$ mean "the dimension" of mass. All other variables in fluid mechanics can be expressed in terms of $\{M\}$, $\{L\}$, $\{T\}$, and $\{\Theta\}$. For example, acceleration has the dimensions $\{LT^{-2}\}$. The most crucial of these secondary dimensions is force, which is directly related to mass, length, and time by Newton's second law

$$\mathbf{F} = m\mathbf{a} \tag{1.2}$$

From this we see that, dimensionally, $\{F\} = \{MLT^{-2}\}$. A constant of proportionality is avoided by defining the force unit exactly in terms of the primary units. Thus we define the newton and the pound of force

$$\begin{aligned} 1 \text{ newton of force} &= 1 \text{ N} \equiv 1 \text{ kg} \cdot \text{m/s}^2 \\ 1 \text{ pound of force} &= 1 \text{ lbf} \equiv 1 \text{ slug} \cdot \text{ft/s}^2 = 4.4482 \text{ N} \end{aligned} \tag{1.3}$$

In this book the abbreviation *lbf* is used for pound-force and *lb* for pound-mass. If instead one adopts other force units such as the dyne or the poundal or kilopond or adopts other mass units such as the gram or pound-mass, a constant of proportionality called g_c must be included in Eq. (1.2). We shall not use g_c in this book since it is not necessary in the SI and BG systems.

A list of some important secondary variables in fluid mechanics, with dimensions derived as combinations of the four primary dimensions, is given in Table 1.2. A more complete list of conversion factors is given in App. C.

Table 1.1 Primary Dimensions in SI and BG Systems

Primary dimension	SI unit	BG unit	Conversion factor
Mass $\{M\}$	Kilogram (kg)	Slug	1 slug = 14.5939 kg
Length $\{L\}$	Meter (m)	Foot (ft)	1 ft = 0.3048 m
Time $\{T\}$	Second (s)	Second (s)	1 s = 1 s
Temperature $\{\Theta\}$	Kelvin (K)	Rankine (°R)	1 K = 1.8°R

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[4] If electromagnetic effects are important, a fifth primary dimension must be included, electric current $\{I\}$, whose SI unit is the ampere (A).

Table 1.2 Secondary Dimensions in Fluid Mechanics

Secondary dimension	SI unit	BG unit	Conversion factor
Area $\{L^2\}$	m^2	ft^2	$1\ m^2 = 10.764\ ft^2$
Volume $\{L^3\}$	m^3	ft^3	$1\ m^3 = 35.315\ ft^3$
Velocity $\{LT^{-1}\}$	m/s	ft/s	1 ft/s = 0.3048 m/s
Acceleration $\{LT^{-2}\}$	m/s^2	ft/s^2	$1\ ft/s^2 = 0.3048\ m/s^2$
Pressure or stress $\{ML^{-1}T^{-2}\}$	$Pa = N/m^2$	lbf/ft^2	$1\ lbf/ft^2 = 47.88\ Pa$
Angular velocity $\{T^{-1}\}$	s^{-1}	s^{-1}	$1\ s^{-1} = 1\ s^{-1}$
Energy, heat, work $\{ML^2T^{-2}\}$	J = N · m	ft · lbf	1 ft · lbf = 1.3558 J
Power $\{ML^2T^{-3}\}$	W = J/s	ft · lbf/s	1 ft · lbf/s = 1.3558 W
Density $\{ML^{-3}\}$	kg/m^3	$slugs/ft^3$	$1\ slug/ft^3 = 515.4\ kg/m^3$
Viscosity $\{ML^{-1}T^{-1}\}$	kg/(m · s)	slugs/(ft · s)	1 slug/(ft · s) = 47.88 kg/(m · s)
Specific heat $\{L^2T^{-2}\Theta^{-1}\}$	$m^2/(s^2 \cdot K)$	$ft^2/(s^2 \cdot °R)$	$1\ m^2/(s^2 \cdot K) = 5.980\ ft^2/(s^2 \cdot °R)$

EXAMPLE 1.1

A body weighs 1000 lbf when exposed to a standard earth gravity $g = 32.174\ ft/s^2$. (*a*) What is its mass in kg? (*b*) What will the weight of this body be in N if it is exposed to the moon's standard acceleration $g_{moon} = 1.62\ m/s^2$? (*c*) How fast will the body accelerate if a net force of 400 lbf is applied to it on the moon or on the earth?

Solution

Part (a) Equation (1.2) holds with F = weight and $a = g_{earth}$:

$$F = W = mg = 1000\ \text{lbf} = (m\ \text{slugs})(32.174\ \text{ft/s}^2)$$

or

$$m = \frac{1000}{32.174} = (31.08\ \text{slugs})(14.5939\ \text{kg/slug}) = 453.6\ \text{kg} \qquad \textit{Ans. (a)}$$

The change from 31.08 slugs to 453.6 kg illustrates the proper use of the conversion factor 14.5939 kg/slug.

Part (b) The mass of the body remains 453.6 kg regardless of its location. Equation (1.2) applies with a new value of a and hence a new force

$$F = W_{moon} = mg_{moon} = (453.6\ \text{kg})(1.62\ \text{m/s}^2) = 735\ \text{N} \qquad \textit{Ans. (b)}$$

Part (c) This problem does not involve weight or gravity or position and is simply a direct application of Newton's law with an unbalanced force:

$$F = 400\ \text{lbf} = ma = (31.08\ \text{slugs})(a\ \text{ft/s}^2)$$

or

$$a = \frac{400}{31.08} = 12.43\ \text{ft/s}^2 = 3.79\ \text{m/s}^2 \qquad \textit{Ans. (c)}$$

This acceleration would be the same on the moon or earth or anywhere.

Many data in the literature are reported in inconvenient or arcane units suitable only to some industry or specialty or country. The engineer should convert these data to the SI or BG system before using them. This requires the systematic application of conversion factors, as in the following example.

EXAMPLE 1.2

An early viscosity unit in the cgs system is the poise (abbreviated P), or g/(cm · s), named after J. L. M. Poiseuille, a French physician who performed pioneering experiments in 1840 on water flow in pipes. The viscosity of water (fresh or salt) at 293.16 K = 20°C is approximately $\mu = 0.01$ P. Express this value in (*a*) SI and (*b*) BG units.

Solution

Part (a)

$$\mu = [0.01\ \text{g/(cm}\cdot\text{s)}]\,\frac{1\ \text{kg}}{1000\ \text{g}}\,(100\ \text{cm/m}) = 0.001\ \text{kg/(m}\cdot\text{s)} \qquad \textit{Ans. (a)}$$

Part (b)

$$\mu = [0.001\ \text{kg/(m}\cdot\text{s)}]\,\frac{1\ \text{slug}}{14.59\ \text{kg}}\,(0.3048\ \text{m/ft})$$

$$= 2.09 \times 10^{-5}\ \text{slug/(ft}\cdot\text{s)} \qquad \textit{Ans. (b)}$$

Note: Result (*b*) could have been found directly from (*a*) by dividing (*a*) by the viscosity conversion factor 47.88 listed in Table 1.2.

We repeat our advice: Faced with data in unusual units, convert them immediately to either SI or BG units because (1) it is more professional and (2) theoretical equations in fluid mechanics are *dimensionally consistent* and require no further conversion factors when these two fundamental unit systems are used, as the following example shows.

EXAMPLE 1.3

A useful theoretical equation for computing the relation between pressure, velocity, and altitude in a steady flow of a nearly inviscid, nearly incompressible fluid with negligible heat transfer and shaft work[5] is the *Bernoulli relation,* named after Daniel Bernoulli, who published a hydrodynamics textbook in 1738:

$$p_0 = p + \tfrac{1}{2}\rho V^2 + \rho g Z \qquad (1)$$

where p_0 = stagnation pressure
p = pressure in moving fluid
V = velocity
ρ = density
Z = altitude
g = gravitational acceleration

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[5] That's an awful lot of assumptions, which need further study in Chap. 3.

(*a*) Show that Eq. (1) satisfies the principle of dimensional homogeneity, which states that all additive terms in a physical equation must have the same dimensions. (*b*) Show that consistent units result without additional conversion factors in SI units. (*c*) Repeat (*b*) for BG units.

Solution

Part (a) We can express Eq. (1) dimensionally, using braces by entering the dimensions of each term from Table 1.2:

$$\{ML^{-1}T^{-2}\} = \{ML^{-1}T^{-2}\} + \{ML^{-3}\}\{L^2T^{-2}\} + \{ML^{-3}\}\{LT^{-2}\}\{L\}$$

$$= \{ML^{-1}T^{-2}\} \text{ for all terms} \qquad \textit{Ans. (a)}$$

Part (b) Enter the SI units for each quantity from Table 1.2:

$$\{\text{N/m}^2\} = \{\text{N/m}^2\} + \{\text{kg/m}^3\}\{\text{m}^2/\text{s}^2\} + \{\text{kg/m}^3\}\{\text{m/s}^2\}\{\text{m}\}$$

$$= \{\text{N/m}^2\} + \{\text{kg/(m}\cdot\text{s}^2)\}$$

The right-hand side looks bad until we remember from Eq. (1.3) that 1 kg = 1 N · s^2/m.

$$\{\text{kg/(m}\cdot\text{s}^2)\} = \frac{\{\text{N}\cdot\text{s}^2/\text{m}\}}{\{\text{m}\cdot\text{s}^2\}} = \{\text{N/m}^2\} \qquad \textit{Ans. (b)}$$

Thus all terms in Bernoulli's equation will have units of pascals, or newtons per square meter, when SI units are used. No conversion factors are needed, which is true of all theoretical equations in fluid mechanics.

Part (c) Introducing BG units for each term, we have

$$\{\text{lbf/ft}^2\} = \{\text{lbf/ft}^2\} + \{\text{slugs/ft}^3\}\{\text{ft}^2/\text{s}^2\} + \{\text{slugs/ft}^3\}\{\text{ft/s}^2\}\{\text{ft}\}$$

$$= \{\text{lbf/ft}^2\} + \{\text{slugs/(ft}\cdot\text{s}^2)\}$$

But, from Eq. (1.3), 1 slug = 1 lbf · s^2/ft, so that

$$\{\text{slugs/(ft}\cdot\text{s}^2)\} = \frac{\{\text{lbf}\cdot\text{s}^2/\text{ft}\}}{\{\text{ft}\cdot\text{s}^2\}} = \{\text{lbf/ft}^2\} \qquad \textit{Ans. (c)}$$

All terms have the unit of pounds-force per square foot. No conversion factors are needed in the BG system either.

There is still a tendency in English-speaking countries to use pound-force per square inch as a pressure unit because the numbers are more manageable. For example, standard atmospheric pressure is 14.7 lbf/in^2 = 2116 lbf/ft^2 = 101,300 Pa. The pascal is a small unit because the newton is less than $\frac{1}{4}$ lbf and a square meter is a very large area. It is felt nevertheless that the pascal will gradually gain universal acceptance; e.g., repair manuals for U.S. automobiles now specify pressure measurements in pascals.

Consistent Units

Note that not only must all (fluid) mechanics equations be dimensionally homogeneous, one must also use *consistent units;* that is, each additive term must have the same units. There is no trouble doing this with the SI and BG systems, as in Ex. 1.3, but woe unto those who try to mix colloquial English units. For example, in Chap. 9, we often use the assumption of steady adiabatic compressible gas flow:

$$h + \tfrac{1}{2}V^2 = \text{constant}$$

where h is the fluid enthalpy and $V^2/2$ is its kinetic energy. Colloquial thermodynamic tables might list h in units of British thermal units per pound (Btu/lb), whereas V is likely used in ft/s. It is completely erroneous to add Btu/lb to ft^2/s^2. The proper unit for h in this case is ft · lbf/slug, which is identical to ft^2/s^2. The conversion factor is 1 Btu/lb $\approx$ 25,040 ft^2/s^2 = 25,040 ft · lbf/slug.

Homogeneous versus Dimensionally Inconsistent Equations

All theoretical equations in mechanics (and in other physical sciences) are *dimensionally homogeneous*; i.e., each additive term in the equation has the same dimensions. For example, Bernoulli's equation (1) in Example 1.3 is dimensionally homogeneous: Each term has the dimensions of pressure or stress of $\{F/L^2\}$. Another example is the equation from physics for a body falling with negligible air resistance:

$$S = S_0 + V_0 t + \tfrac{1}{2}gt^2$$

where S_0 is initial position, V_0 is initial velocity, and g is the acceleration of gravity. Each term in this relation has dimensions of length $\{L\}$. The factor $\frac{1}{2}$, which arises from integration, is a pure (dimensionless) number, $\{1\}$. The exponent 2 is also dimensionless.

However, the reader should be warned that many empirical formulas in the engineering literature, arising primarily from correlations of data, are dimensionally inconsistent. Their units cannot be reconciled simply, and some terms may contain hidden variables. An example is the formula which pipe valve manufacturers cite for liquid volume flow rate Q (m^3/s) through a partially open valve:

$$Q = C_V\left(\frac{\Delta p}{\text{SG}}\right)^{1/2}$$

where Δp is the pressure drop across the valve and SG is the specific gravity of the liquid (the ratio of its density to that of water). The quantity C_V is the *valve flow coefficient,* which manufacturers tabulate in their valve brochures. Since SG is dimensionless $\{1\}$, we see that this formula is totally inconsistent, with one side being a flow rate $\{L^3/T\}$ and the other being the square root of a pressure drop $\{M^{1/2}/L^{1/2}T\}$. It follows that C_V must have dimensions, and rather odd ones at that: $\{L^{7/2}/M^{1/2}\}$. Nor is the resolution of this discrepancy clear, although one hint is that the values of C_V in the literature increase nearly as the square of the size of the valve. The presentation of experimental data in homogeneous form is the subject of *dimensional analysis* (Chap. 5). There we shall learn that a homogeneous form for the valve flow relation is

$$Q = C_d A_{\text{opening}}\left(\frac{\Delta p}{\rho}\right)^{1/2}$$

where ρ is the liquid density and A the area of the valve opening. The *discharge coefficient* C_d is dimensionless and changes only slightly with valve size. Please believe—until we establish the fact in Chap. 5—that this latter is a *much* better formulation of the data.

Meanwhile, we conclude that dimensionally inconsistent equations, though they abound in engineering practice, are misleading and vague and even dangerous, in the sense that they are often misused outside their range of applicability.

Convenient Prefixes in Powers of 10

Engineering results often are too small or too large for the common units, with too many zeros one way or the other. For example, to write $p = 114{,}000{,}000$ Pa is long and awkward. Using the prefix "M" to mean 10^6, we convert this to a concise $p = 114$ MPa (megapascals). Similarly, $t = 0.000000003$ s is a proofreader's nightmare compared to the equivalent $t = 3$ ns (nanoseconds). Such prefixes are common and convenient, in both the SI and BG systems. A complete list is given in Table 1.3.

Table 1.3 Convenient Prefixes for Engineering Units

Multiplicative factor	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deka	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	milli	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

EXAMPLE 1.4

In 1890 Robert Manning, an Irish engineer, proposed the following empirical formula for the average velocity V in uniform flow due to gravity down an open channel (BG units):

$$V = \frac{1.49}{n} R^{2/3} S^{1/2} \qquad (1)$$

where R = hydraulic radius of channel (Chaps. 6 and 10)
S = channel slope (tangent of angle that bottom makes with horizontal)
n = Manning's roughness factor (Chap. 10)

and n is a constant for a given surface condition for the walls and bottom of the channel. (*a*) Is Manning's formula dimensionally consistent? (*b*) Equation (1) is commonly taken to be valid in BG units with n taken as dimensionless. Rewrite it in SI form.

Solution

Part (a) Introduce dimensions for each term. The slope S, being a tangent or ratio, is dimensionless, denoted by {unity} or {1}. Equation (1) in dimensional form is

$$\left\{\frac{L}{T}\right\} = \left\{\frac{1.49}{n}\right\}\{L^{2/3}\}\{1\}$$

This formula cannot be consistent unless $\{1.49/n\} = \{L^{1/3}/T\}$. If n is dimensionless (and it is never listed with units in textbooks), then the numerical value 1.49 must have units. This can be tragic to an engineer working in a different unit system unless the discrepancy is properly documented. In fact, Manning's formula, though popular, is inconsistent both dimensionally and physically and does not properly account for channel-roughness effects except in a narrow range of parameters, for water only.

Part (b) From part (*a*), the number 1.49 must have dimensions $\{L^{1/3}/T\}$ and thus in BG units equals 1.49 $\text{ft}^{1/3}$/s. By using the SI conversion factor for length we have

$$(1.49\ \text{ft}^{1/3}/\text{s})(0.3048\ \text{m/ft})^{1/3} = 1.00\ \text{m}^{1/3}/\text{s}$$

Therefore Manning's formula in SI becomes

$$V = \frac{1.0}{n} R^{2/3} S^{1/2} \qquad \textit{Ans. (b)}\ (2)$$

with R in m and V in m/s. Actually, we misled you: This is the way Manning, a metric user, first proposed the formula. It was later converted to BG units. Such dimensionally inconsistent formulas are dangerous and should either be reanalyzed or treated as having very limited application.

1.5 Properties of the Velocity Field

In a given flow situation, the determination, by experiment or theory, of the properties of the fluid as a function of position and time is considered to be the *solution* to the problem. In almost all cases, the emphasis is on the space-time distribution of the fluid properties. One rarely keeps track of the actual fate of the specific fluid particles.[6] This treatment of properties as continuum-field functions distinguishes fluid mechanics from solid mechanics, where we are more likely to be interested in the trajectories of individual particles or systems.

Eulerian and Lagrangian Descriptions

There are two different points of view in analyzing problems in mechanics. The first view, appropriate to fluid mechanics, is concerned with the field of flow and is called the *eulerian* method of description. In the eulerian method we compute the pressure field $p(x, y, z, t)$ of the flow pattern, not the pressure changes $p(t)$ which a particle experiences as it moves through the field.

The second method, which follows an individual particle moving through the flow, is called the *lagrangian* description. The lagrangian approach, which is more appropriate to solid mechanics, will not be treated in this book. However, certain numerical analyses of sharply bounded fluid flows, such as the motion of isolated fluid droplets, are very conveniently computed in lagrangian coordinates [1].

Fluid-dynamic measurements are also suited to the eulerian system. For example, when a pressure probe is introduced into a laboratory flow, it is fixed at a specific position (x, y, z). Its output thus contributes to the description of the eulerian pressure field $p(x, y, z, t)$. To simulate a lagrangian measurement, the probe would have to move downstream at the fluid particle speeds; this is sometimes done in oceanographic measurements, where flowmeters drift along with the prevailing currents.

The two different descriptions can be contrasted in the analysis of traffic flow along a freeway. A certain length of freeway may be selected for study and called the field of flow. Obviously, as time passes, various cars will enter and leave the field, and the identity of the specific cars within the field will constantly be changing. The traffic engineer ignores specific cars and concentrates on their average velocity as a function of time and position within the field, plus the flow rate or number of cars per hour passing a given section of the freeway. This engineer is using an eulerian description of the traffic flow. Other investigators, such as the police or social scientists, may be interested in the path or speed or destination of specific cars in the field. By following a specific car as a function of time, they are using a lagrangian description of the flow.

The Velocity Field

Foremost among the properties of a flow is the velocity field $\mathbf{V}(x, y, z, t)$. In fact, determining the velocity is often tantamount to solving a flow problem, since other prop-

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[6] One example where fluid-particle paths are important is in water-quality analysis of the fate of contaminant discharges.

erties follow directly from the velocity field. Chapter 2 is devoted to the calculation of the pressure field once the velocity field is known. Books on heat transfer (for example, Ref. 10) are essentially devoted to finding the temperature field from known velocity fields.

In general, velocity is a vector function of position and time and thus has three components u, v, and w, each a scalar field in itself:

$$\mathbf{V}(x, y, z, t) = \mathbf{i}u(x, y, z, t) + \mathbf{j}v(x, y, z, t) + \mathbf{k}w(x, y, z, t) \tag{1.4}$$

The use of u, v, and w instead of the more logical component notation V_x, V_y, and V_z is the result of an almost unbreakable custom in fluid mechanics.

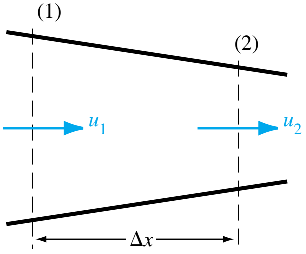
Several other quantities, called *kinematic properties,* can be derived by mathematically manipulating the velocity field. We list some kinematic properties here and give more details about their use and derivation in later chapters:

1. Displacement vector: $\mathbf{r} = \int \mathbf{V}\, dt$ (Sec. 1.9)
2. Acceleration: $\mathbf{a} = \dfrac{d\mathbf{V}}{dt}$ (Sec. 4.1)
3. Volume rate of flow: $Q = \int (\mathbf{V} \cdot \mathbf{n})\, dA$ (Sec. 3.2)
4. Volume expansion rate: $\dfrac{1}{\mathcal{V}} \dfrac{d\mathcal{V}}{dt} = \nabla \cdot \mathbf{V}$ (Sec. 4.2)
5. Local angular velocity: $\omega = \frac{1}{2}\nabla \times \mathbf{V}$ (Sec. 4.8)

We will not illustrate any problems regarding these kinematic properties at present. The point of the list is to illustrate the type of vector operations used in fluid mechanics and to make clear the dominance of the velocity field in determining other flow properties. *Note:* The fluid *acceleration,* item 2 above, is not as simple as it looks and actually involves four different terms due to the use of the chain rule in calculus (see Sec. 4.1).

EXAMPLE 1.5

Fluid flows through a contracting section of a duct, as in Fig. E1.5. A velocity probe inserted at section (1) measures a steady value $u_1 = 1$ m/s, while a similar probe at section (2) records a steady $u_2 = 3$ m/s. Estimate the fluid acceleration, if any, if $\Delta x = 10$ cm.



E1.5

Solution

The flow is *steady* (not time-varying), but fluid particles clearly increase in velocity as they pass from (1) to (2). This is the concept of *convective acceleration* (Sec. 4.1). We may estimate the acceleration as a velocity change Δu divided by a time change $\Delta t = \Delta x / u_{\text{avg}}$:

$$a_x \approx \frac{\text{velocity change}}{\text{time change}} \approx \frac{u_2 - u_1}{\Delta x / [\frac{1}{2}(u_1 + u_2)]} = \frac{(3.0 - 1.0 \text{ m/s})(1.0 + 3.0 \text{ m/s})}{2(0.1 \text{ m})} \approx 40 \text{ m/s}^2 \qquad \textit{Ans.}$$

A simple estimate thus indicates that this seemingly innocuous flow is accelerating at 4 times the acceleration of gravity. In the limit as Δx and Δt become very small, the above estimate reduces to a partial-derivative expression for convective x-acceleration:

$$a_{x,\text{convective}} = \lim_{\Delta t \to 0} \frac{\Delta u}{\Delta t} = u\frac{\partial u}{\partial x}$$

In three-dimensional flow (Sec. 4.1) there are *nine* of these convective terms.

1.6 Thermodynamic Properties of a Fluid

While the velocity field **V** is the most important fluid property, it interacts closely with the thermodynamic properties of the fluid. We have already introduced into the discussion the three most common such properties

1. Pressure p
2. Density ρ
3. Temperature T

These three are constant companions of the velocity vector in flow analyses. Four other thermodynamic properties become important when work, heat, and energy balances are treated (Chaps. 3 and 4):

4. Internal energy e
5. Enthalpy $h = \hat{u} + p/\rho$
6. Entropy s
7. Specific heats c_p and c_v

In addition, friction and heat conduction effects are governed by the two so-called *transport properties:*

8. Coefficient of viscosity μ
9. Thermal conductivity k

All nine of these quantities are true thermodynamic properties which are determined by the thermodynamic condition or *state* of the fluid. For example, for a single-phase substance such as water or oxygen, two basic properties such as pressure and temperature are sufficient to fix the value of all the others:

$$\rho = \rho(p, T) \qquad h = h(p, T) \qquad \mu = \mu(p, T) \tag{1.5}$$

and so on for every quantity in the list. Note that the specific volume, so important in thermodynamic analyses, is omitted here in favor of its inverse, the density ρ.

Recall that thermodynamic properties describe the state of a *system,* i.e., a collection of matter of fixed identity which interacts with its surroundings. In most cases here the system will be a small fluid element, and all properties will be assumed to be continuum properties of the flow field: $\rho = \rho(x, y, z, t)$, etc.

Recall also that thermodynamics is normally concerned with *static* systems, whereas fluids are usually in variable motion with constantly changing properties. Do the properties retain their meaning in a fluid flow which is technically not in equilibrium? The answer is yes, from a statistical argument. In gases at normal pressure (and even more so for liquids), an enormous number of molecular collisions occur over a very short distance of the order of 1 μm, so that a fluid subjected to sudden changes rapidly adjusts itself toward equilibrium. We therefore assume that all the thermodynamic properties listed above exist as point functions in a flowing fluid and follow all the laws and state relations of ordinary equilibrium thermodynamics. There are, of course, important nonequilibrium effects such as chemical and nuclear reactions in flowing fluids which are not treated in this text.

Pressure

Pressure is the (compression) stress at a point in a static fluid (Fig. 1.1). Next to velocity, the pressure p is the most dynamic variable in fluid mechanics. Differences or *gradients* in pressure often drive a fluid flow, especially in ducts. In low-speed flows, the actual magnitude of the pressure is often not important, unless it drops so low as to cause vapor bubbles to form in a liquid. For convenience, we set many such problem assignments at the level of 1 atm $= 2116$ lbf/ft^2 $= 101{,}300$ Pa. High-speed (compressible) gas flows (Chap. 9), however, are indeed sensitive to the magnitude of pressure.

Temperature

Temperature T is a measure of the internal energy level of a fluid. It may vary considerably during high-speed flow of a gas (Chap. 9). Although engineers often use Celsius or Fahrenheit scales for convenience, many applications in this text require *absolute* (Kelvin or Rankine) temperature scales:

$$
\begin{aligned}
{}^\circ\mathrm{R} &= {}^\circ\mathrm{F} + 459.69 \\
\mathrm{K} &= {}^\circ\mathrm{C} + 273.16
\end{aligned}
$$

If temperature differences are strong, *heat transfer* may be important [10], but our concern here is mainly with dynamic effects. We examine heat-transfer principles briefly in Secs. 4.5 and 9.8.

Density

The density of a fluid, denoted by ρ (lowercase Greek rho), is its mass per unit volume. Density is highly variable in gases and increases nearly proportionally to the pressure level. Density in liquids is nearly constant; the density of water (about 1000 kg/m^3) increases only 1 percent if the pressure is increased by a factor of 220. Thus most liquid flows are treated analytically as nearly "incompressible."

In general, liquids are about three orders of magnitude more dense than gases at atmospheric pressure. The heaviest common liquid is mercury, and the lightest gas is hydrogen. Compare their densities at 20°C and 1 atm:

$$\text{Mercury:} \quad \rho = 13{,}580 \text{ kg/m}^3 \qquad \text{Hydrogen:} \quad \rho = 0.0838 \text{ kg/m}^3$$

They differ by a factor of 162,000! Thus the physical parameters in various liquid and gas flows might vary considerably. The differences are often resolved by the use of *dimensional analysis* (Chap. 5). Other fluid densities are listed in Tables A.3 and A.4 (in App. A).

Specific Weight

The *specific weight* of a fluid, denoted by γ (lowercase Greek gamma), is its weight per unit volume. Just as a mass has a weight $W = mg$, density and specific weight are simply related by gravity:

$$\gamma = \rho g \tag{1.6}$$

The units of γ are weight per unit volume, in lbf/ft^3 or N/m^3. In standard earth gravity, $g = 32.174$ ft/s^2 $= 9.807$ m/s^2. Thus, e.g., the specific weights of air and water at 20°C and 1 atm are approximately

$$\gamma_{\text{air}} = (1.205 \text{ kg/m}^3)(9.807 \text{ m/s}^2) = 11.8 \text{ N/m}^3 = 0.0752 \text{ lbf/ft}^3$$

$$\gamma_{\text{water}} = (998 \text{ kg/m}^3)(9.807 \text{ m/s}^2) = 9790 \text{ N/m}^3 = 62.4 \text{ lbf/ft}^3$$

Specific weight is very useful in the hydrostatic-pressure applications of Chap. 2. Specific weights of other fluids are given in Tables A.3 and A.4.

Specific Gravity

Specific gravity, denoted by SG, is the ratio of a fluid density to a standard reference fluid, water (for liquids), and air (for gases):

$$\text{SG}_{\text{gas}} = \frac{\rho_{\text{gas}}}{\rho_{\text{air}}} = \frac{\rho_{\text{gas}}}{1.205 \text{ kg/m}^3} \tag{1.7}$$

$$\text{SG}_{\text{liquid}} = \frac{\rho_{\text{liquid}}}{\rho_{\text{water}}} = \frac{\rho_{\text{liquid}}}{998 \text{ kg/m}^3}$$

For example, the specific gravity of mercury (Hg) is $\text{SG}_{\text{Hg}} = 13{,}580/998 \approx 13.6$. Engineers find these dimensionless ratios easier to remember than the actual numerical values of density of a variety of fluids.

Potential and Kinetic Energies

In thermostatics the only energy in a substance is that stored in a system by molecular activity and molecular bonding forces. This is commonly denoted as *internal energy* $\hat{u}$. A commonly accepted adjustment to this static situation for fluid flow is to add two more energy terms which arise from newtonian mechanics: the potential energy and kinetic energy.

The potential energy equals the work required to move the system of mass m from the origin to a position vector $\mathbf{r} = \mathbf{i}x + \mathbf{j}y + \mathbf{k}z$ against a gravity field $\mathbf{g}$. Its value is $-m\mathbf{g} \cdot \mathbf{r}$, or $-\mathbf{g} \cdot \mathbf{r}$ per unit mass. The kinetic energy equals the work required to change the speed of the mass from zero to velocity V. Its value is $\frac{1}{2}mV^2$ or $\frac{1}{2}V^2$ per unit mass. Then by common convention the total stored energy e per unit mass in fluid mechanics is the sum of three terms:

$$e = \hat{u} + \tfrac{1}{2}V^2 + (-\mathbf{g} \cdot \mathbf{r}) \tag{1.8}$$

Also, throughout this book we shall define z as upward, so that $\mathbf{g} = -g\mathbf{k}$ and $\mathbf{g} \cdot \mathbf{r} = -gz$. Then Eq. (1.8) becomes

$$e = \hat{u} + \tfrac{1}{2}V^2 + gz \tag{1.9}$$

The molecular internal energy $\hat{u}$ is a function of T and p for the single-phase pure substance, whereas the potential and kinetic energies are kinematic properties.

State Relations for Gases

Thermodynamic properties are found both theoretically and experimentally to be related to each other by state relations which differ for each substance. As mentioned, we shall confine ourselves here to single-phase pure substances, e.g., water in its liquid phase. The second most common fluid, air, is a mixture of gases, but since the mixture ratios remain nearly constant between 160 and 2200 K, in this temperature range air can be considered to be a pure substance.

All gases at high temperatures and low pressures (relative to their critical point) are in good agreement with the *perfect-gas law*

$$p = \rho R T \qquad R = c_p - c_v = \text{gas constant} \tag{1.10}$$

Since Eq. (1.10) is dimensionally consistent, R has the same dimensions as specific heat, $\{L^2T^{-2}\Theta^{-1}\}$, or velocity squared per temperature unit (kelvin or degree Rankine). Each gas has its own constant R, equal to a universal constant Λ divided by the molecular weight

$$R_{\text{gas}} = \frac{\Lambda}{M_{\text{gas}}} \tag{1.11}$$

where $\Lambda = 49{,}700\ \text{ft}^2/(\text{s}^2 \cdot {}^\circ\text{R}) = 8314\ \text{m}^2/(\text{s}^2 \cdot \text{K})$. Most applications in this book are for air, with $M = 28.97$:

$$R_{\text{air}} = 1717\ \text{ft}^2/(\text{s}^2 \cdot {}^\circ\text{R}) = 287\ \text{m}^2/(\text{s}^2 \cdot \text{K}) \tag{1.12}$$

Standard atmospheric pressure is 2116 lbf/ft^2, and standard temperature is 60°F = 520°R. Thus standard air density is

$$\rho_{\text{air}} = \frac{2116}{(1717)(520)} = 0.00237\ \text{slug/ft}^3 = 1.22\ \text{kg/m}^3 \tag{1.13}$$

This is a nominal value suitable for problems.

One proves in thermodynamics that Eq. (1.10) requires that the internal molecular energy $\hat{u}$ of a perfect gas vary only with temperature: $\hat{u} = \hat{u}(T)$. Therefore the specific heat c_v also varies only with temperature:

$$c_v = \left(\frac{\partial \hat{u}}{\partial T}\right)_\rho = \frac{d\hat{u}}{dT} = c_v(T)$$

or

$$d\hat{u} = c_v(T)\, dT \tag{1.14}$$

In like manner h and c_p of a perfect gas also vary only with temperature:

$$h = \hat{u} + \frac{p}{\rho} = \hat{u} + RT = h(T)$$

$$c_p = \left(\frac{\partial h}{\partial T}\right)_p = \frac{dh}{dT} = c_p(T) \tag{1.15}$$

$$dh = c_p(T)\, dT$$

The ratio of specific heats of a perfect gas is an important dimensionless parameter in compressible-flow analysis (Chap. 9)

$$k = \frac{c_p}{c_v} = k(T) \geq 1 \tag{1.16}$$

As a first approximation in airflow analysis we commonly take c_p, c_v, and k to be constant

$$k_{\text{air}} \approx 1.4$$

$$c_v = \frac{R}{k-1} \approx 4293 \text{ ft}^2/(\text{s}^2 \cdot {}^\circ\text{R}) = 718 \text{ m}^2/(\text{s}^2 \cdot \text{K})$$

$$c_p = \frac{kR}{k-1} \approx 6010 \text{ ft}^2/(\text{s}^2 \cdot {}^\circ\text{R}) = 1005 \text{ m}^2/(\text{s}^2 \cdot \text{K}) \tag{1.17}$$

Actually, for all gases, c_p and c_v increase gradually with temperature, and k decreases gradually. Experimental values of the specific-heat ratio for eight common gases are shown in Fig. 1.3.

Many flow problems involve steam. Typical steam operating conditions are relatively close to the critical point, so that the perfect-gas approximation is inaccurate. The properties of steam are therefore available in tabular form [13], but the error of using the perfect-gas law is sometimes not great, as the following example shows.

EXAMPLE 1.6

Estimate ρ and c_p of steam at 100 lbf/in^2 and 400°F (*a*) by a perfect-gas approximation and (*b*) from the ASME steam tables [13].

Solution

Part (a) First convert to BG units: $p = 100 \text{ lbf/in}^2 = 14{,}400 \text{ lb/ft}^2$, $T = 400^\circ\text{F} = 860^\circ\text{R}$. From Table A.4 the molecular weight of H_2O is $2M_H + M_O = 2(1.008) + 16.0 = 18.016$. Then from Eq. (1.11) the gas constant of steam is approximately

$$R = \frac{49{,}700}{18.016} = 2759 \text{ ft}^2/(\text{s}^2 \cdot {}^\circ\text{R})$$

whence, from the perfect-gas law,

$$\rho \approx \frac{p}{RT} = \frac{14{,}400}{2759(860)} = 0.00607 \text{ slug/ft}^3 \qquad \textit{Ans. (a)}$$

From Fig. 1.3, k for steam at 860°R is approximately 1.30. Then from Eq. (1.17),

$$c_p \approx \frac{kR}{k-1} = \frac{1.30(2759)}{1.30-1} = 12{,}000 \text{ ft}^2/(\text{s}^2 \cdot {}^\circ\text{R}) \qquad \textit{Ans. (a)}$$

Part (b) From Ref. 13, the specific volume v of steam at 100 lbf/in^2 and 400°F is 4.935 ft^3/lbm. Then the density is the inverse of this, converted to slugs:

$$\rho = \frac{1}{v} = \frac{1}{(4.935 \text{ ft}^2/\text{lbm})(32.174 \text{ lbm/slug})} = 0.00630 \text{ slug/ft}^3 \qquad \textit{Ans. (b)}$$

This is about 4 percent higher than our ideal-gas estimate in part (*a*).

Reference 13 lists the value of c_p of steam at 100 lbf/in^2 and 400°F as 0.535 Btu/(lbm · °F). Convert this to BG units:

$$\begin{aligned} c_p &= [0.535 \text{ Btu/(lbm} \cdot {}^\circ\text{R)}](778.2 \text{ ft} \cdot \text{lbf/Btu})(32.174 \text{ lbm/slug}) \\ &= 13{,}400 \text{ ft} \cdot \text{lbf/(slug} \cdot {}^\circ\text{R)} = 13{,}400 \text{ ft}^2/(\text{s}^2 \cdot {}^\circ\text{R}) \qquad \textit{Ans. (b)} \end{aligned}$$

This is about 11 percent higher than our ideal-gas estimate in part (*a*). The chief reason for the discrepancy is that this temperature and this pressure are quite close to the critical point and saturation line of steam. At higher temperatures and lower pressures, say, 800°F and 50 lbf/in^2, the perfect-gas law gives ρ and c_p of steam within an accuracy of ±1 percent.

Note that the use of pound-mass and British thermal units in the traditional steam tables requires continual awkward conversions to BG units. Newer tables and disks are in SI units.

State Relations for Liquids

The writer knows of no "perfect-liquid law" comparable to that for gases. Liquids are nearly incompressible and have a single reasonably constant specific heat. Thus an idealized state relation for a liquid is

$$\rho \approx \text{const} \qquad c_p \approx c_v \approx \text{const} \qquad dh \approx c_p\, dT \tag{1.18}$$

Most of the flow problems in this book can be attacked with these simple assumptions. Water is normally taken to have a density of 1.94 slugs/ft^3 and a specific heat c_p = 25,200 ft^2/(s^2 · °R). The steam tables may be used if more accuracy is required.

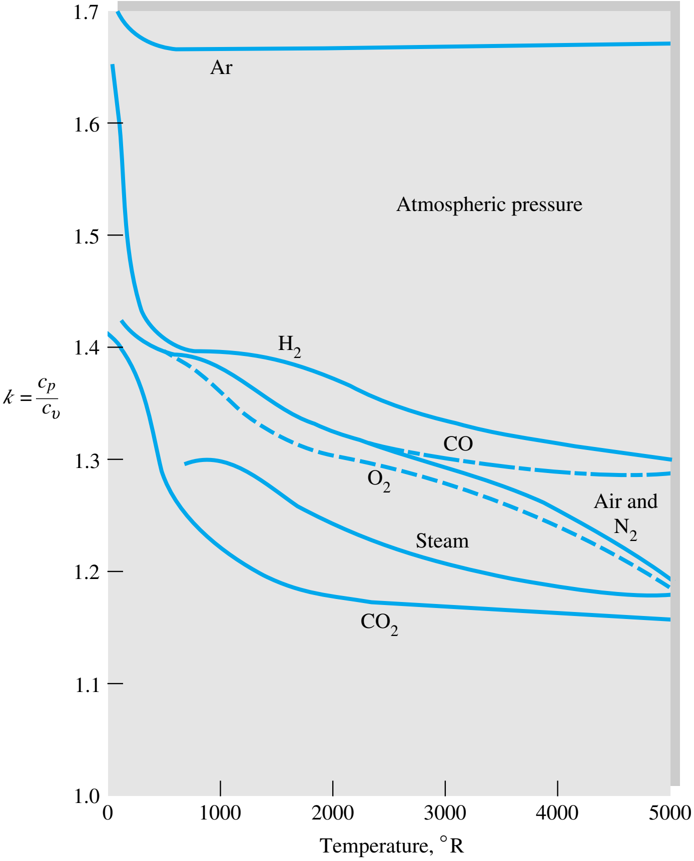


Fig. 1.3 Specific-heat ratio of eight common gases as a function of temperature. *(Data from Ref. 12.)*

The density of a liquid usually decreases slightly with temperature and increases moderately with pressure. If we neglect the temperature effect, an empirical pressure-density relation for a liquid is

$$\frac{p}{p_a} \approx (B + 1)\left(\frac{\rho}{\rho_a}\right)^n - B \tag{1.19}$$

where B and n are dimensionless parameters which vary slightly with temperature and p_a and ρ_a are standard atmospheric values. Water can be fitted approximately to the values $B \approx 3000$ and $n \approx 7$.

Seawater is a variable mixture of water and salt and thus requires three thermodynamic properties to define its state. These are normally taken as pressure, temperature, and the *salinity* $\hat{S}$, defined as the weight of the dissolved salt divided by the weight of the mixture. The average salinity of seawater is 0.035, usually written as 35 parts per 1000, or 35 ‰. The average density of seawater is 2.00 slugs/ft^3. Strictly speaking, seawater has three specific heats, all approximately equal to the value for pure water of 25,200 ft^2/(s^2 · °R) = 4210 m^2/(s^2 · K).

EXAMPLE 1.7

The pressure at the deepest part of the ocean is approximately 1100 atm. Estimate the density of seawater at this pressure.

Solution

Equation (1.19) holds for either water or seawater. The ratio p/p_a is given as 1100:

$$1100 \approx (3001)\left(\frac{\rho}{\rho_a}\right)^7 - 3000$$

or

$$\frac{\rho}{\rho_a} = \left(\frac{4100}{3001}\right)^{1/7} = 1.046$$

Assuming an average surface seawater density $\rho_a = 2.00$ slugs/ft^3, we compute

$$\rho \approx 1.046(2.00) = 2.09 \text{ slugs/ft}^3 \qquad \textit{Ans.}$$

Even at these immense pressures, the density increase is less than 5 percent, which justifies the treatment of a liquid flow as essentially incompressible.

1.7 Viscosity and Other Secondary Properties

The quantities such as pressure, temperature, and density discussed in the previous section are *primary* thermodynamic variables characteristic of any system. There are also certain secondary variables which characterize specific fluid-mechanical behavior. The most important of these is viscosity, which relates the local stresses in a moving fluid to the strain rate of the fluid element.

Viscosity

When a fluid is sheared, it begins to move at a strain rate inversely proportional to a property called its *coefficient of viscosity* μ. Consider a fluid element sheared in one plane by a single shear stress τ, as in Fig. 1.4*a*. The shear strain angle $\delta\theta$ will continuously grow with time as long as the stress τ is maintained, the upper surface moving at speed δu larger than the lower. Such common fluids as water, oil, and air show a linear relation between applied shear and resulting strain rate

$$\tau \propto \frac{\delta\theta}{\delta t} \tag{1.20}$$

From the geometry of Fig. 1.4*a* we see that

$$\tan\,\delta\theta = \frac{\delta u\;\delta t}{\delta y} \tag{1.21}$$

In the limit of infinitesimal changes, this becomes a relation between shear strain rate and velocity gradient

$$\frac{d\theta}{dt} = \frac{du}{dy} \tag{1.22}$$

From Eq. (1.20), then, the applied shear is also proportional to the velocity gradient for the common linear fluids. The constant of proportionality is the viscosity coefficient μ

$$\tau = \mu\frac{d\theta}{dt} = \mu\frac{du}{dy} \tag{1.23}$$

Equation (1.23) is dimensionally consistent; therefore μ has dimensions of stress-time: $\{FT/L^2\}$ or $\{M/(LT)\}$. The BG unit is slugs per foot-second, and the SI unit is kilograms per meter-second. The linear fluids which follow Eq. (1.23) are called *newtonian fluids,* after Sir Isaac Newton, who first postulated this resistance law in 1687.

We do not really care about the strain angle $\theta(t)$ in fluid mechanics, concentrating instead on the velocity distribution $u(y)$, as in Fig. 1.4*b*. We shall use Eq. (1.23) in Chap. 4 to derive a differential equation for finding the velocity distribution $u(y)$—and, more generally, $\mathbf{V}(x, y, z, t)$—in a viscous fluid. Figure 1.4*b* illustrates a shear layer, or *boundary layer,* near a solid wall. The shear stress is proportional to the slope of the

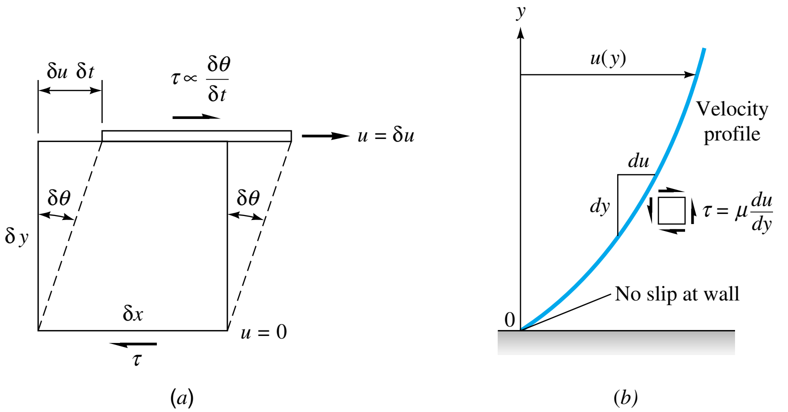


Fig. 1.4 Shear stress causes continuous shear deformation in a fluid: (*a*) a fluid element straining at a rate $\delta\theta/\delta t$; (*b*) newtonian shear distribution in a shear layer near a wall.

Table 1.4 Viscosity and Kinematic Viscosity of Eight Fluids at 1 atm and 20°C

Fluid	μ, kg/(m · s)[†]	Ratio $\mu/\mu(H_2)$	ρ, kg/m^3	ν m^2/s[†]	Ratio $\nu/\nu(Hg)$
Hydrogen	8.8 E−6	1.0	0.084	1.05 E−4	920
Air	1.8 E−5	2.1	1.20	1.51 E−5	130
Gasoline	2.9 E−4	33	680	4.22 E−7	3.7
Water	1.0 E−3	114	998	1.01 E−6	8.7
Ethyl alcohol	1.2 E−3	135	789	1.52 E−6	13
Mercury	1.5 E−3	170	13,580	1.16 E−7	1.0
SAE 30 oil	0.29	33,000	891	3.25 E−4	2,850
Glycerin	1.5	170,000	1,264	1.18 E−3	10,300

[†] 1 kg/(m · s) = 0.0209 slug/(ft · s); 1 m^2/s = 10.76 ft^2/s.

velocity profile and is greatest at the wall. Further, at the wall, the velocity u is zero relative to the wall: This is called the *no-slip condition* and is characteristic of all viscous-fluid flows.

The viscosity of newtonian fluids is a true thermodynamic property and varies with temperature and pressure. At a given state (p, T) there is a vast range of values among the common fluids. Table 1.4 lists the viscosity of eight fluids at standard pressure and temperature. There is a variation of six orders of magnitude from hydrogen up to glycerin. Thus there will be wide differences between fluids subjected to the same applied stresses.

Generally speaking, the viscosity of a fluid increases only weakly with pressure. For example, increasing p from 1 to 50 atm will increase μ of air only 10 percent. Temperature, however, has a strong effect, with μ increasing with T for gases and decreasing for liquids. Figure A.1 (in App. A) shows this temperature variation for various common fluids. It is customary in most engineering work to neglect the pressure variation.

The variation $\mu(p, T)$ for a typical fluid is nicely shown by Fig. 1.5, from Ref. 14, which normalizes the data with the *critical-point state* (μ_c, p_c, T_c). This behavior, called the *principle of corresponding states,* is characteristic of all fluids, but the actual numerical values are uncertain to ±20 percent for any given fluid. For example, values of $\mu(T)$ for air at 1 atm, from Table A.2, fall about 8 percent low compared to the "low-density limit" in Fig. 1.5.

Note in Fig. 1.5 that changes with temperature occur very rapidly near the critical point. In general, critical-point measurements are extremely difficult and uncertain.

The Reynolds Number

As we shall see in Chaps. 5 through 7, the primary parameter correlating the viscous behavior of all newtonian fluids is the dimensionless *Reynolds number:*

$$\mathrm{Re} = \frac{\rho V L}{\mu} = \frac{VL}{\nu} \qquad (1.24)$$

where V and L are characteristic velocity and length scales of the flow. The second form of Re illustrates that the ratio of μ to ρ has its own name, the *kinematic viscosity:*

$$\nu = \frac{\mu}{\rho} \qquad (1.25)$$

It is called kinematic because the mass units cancel, leaving only the dimensions $\{L^2/T\}$.

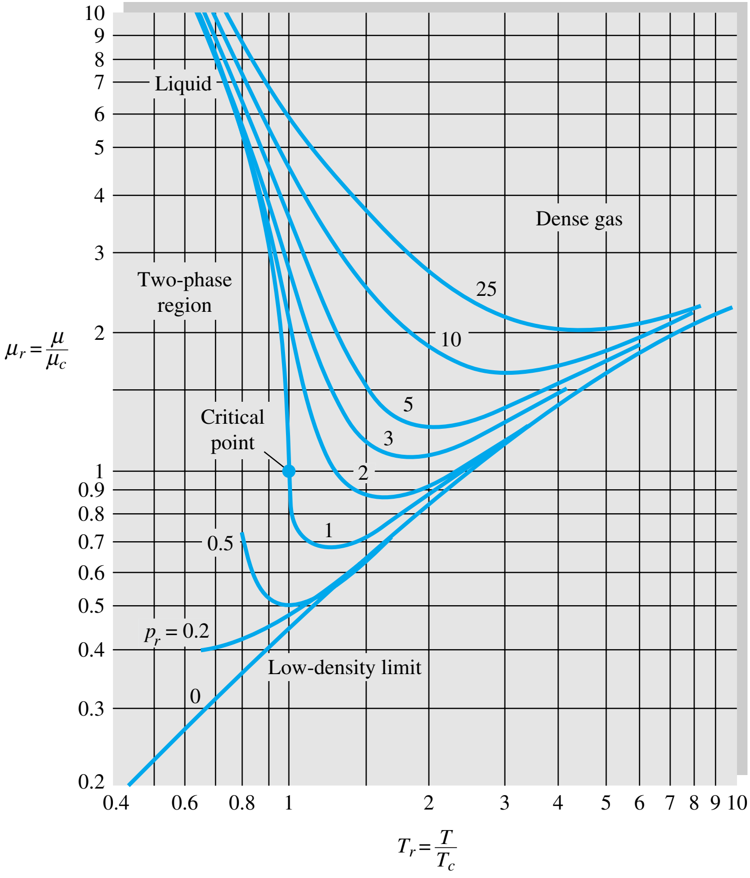


Fig. 1.5 Fluid viscosity nondimensionalized by critical-point properties. This generalized chart is characteristic of all fluids but is only accurate to ±20 percent. *(From Ref. 14.)*

Generally, the first thing a fluids engineer should do is estimate the Reynolds number range of the flow under study. Very low Re indicates viscous *creeping* motion, where inertia effects are negligible. Moderate Re implies a smoothly varying *laminar* flow. High Re probably spells *turbulent* flow, which is slowly varying in the time-mean but has superimposed strong random high-frequency fluctuations. Explicit numerical values for low, moderate, and high Reynolds numbers cannot be stated here. They depend upon flow geometry and will be discussed in Chaps. 5 through 7.

Table 1.4 also lists values of ν for the same eight fluids. The pecking order changes considerably, and mercury, the heaviest, has the smallest viscosity relative to its own weight. All gases have high ν relative to thin liquids such as gasoline, water, and alcohol. Oil and glycerin still have the highest ν, but the ratio is smaller. For a given value of V and L in a flow, these fluids exhibit a spread of four orders of magnitude in the Reynolds number.

Flow between Plates

A classic problem is the flow induced between a fixed lower plate and an upper plate moving steadily at velocity **V**, as shown in Fig. 1.6. The clearance between plates is h, and the fluid is newtonian and does not slip at either plate. If the plates are large,

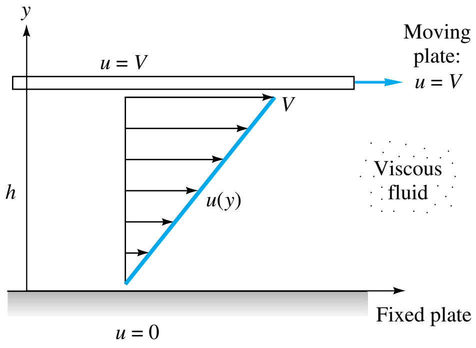


Fig. 1.6 Viscous flow induced by relative motion between two parallel plates.

this steady shearing motion will set up a velocity distribution $u(y)$, as shown, with $v = w = 0$. The fluid acceleration is zero everywhere.

With zero acceleration and assuming no pressure variation in the flow direction, you should show that a force balance on a small fluid element leads to the result that the shear stress is constant throughout the fluid. Then Eq. (1.23) becomes

$$\frac{du}{dy} = \frac{\tau}{\mu} = \text{const}$$

which we can integrate to obtain

$$u = a + by$$

The velocity distribution is linear, as shown in Fig. 1.6, and the constants a and b can be evaluated from the no-slip condition at the upper and lower walls:

$$u = \begin{cases} 0 = a + b(0) & \text{at } y = 0 \\ V = a + b(h) & \text{at } y = h \end{cases}$$

Hence $a = 0$ and $b = V/h$. Then the velocity profile between the plates is given by

$$u = V\frac{y}{h} \tag{1.26}$$

as indicated in Fig. 1.6. Turbulent flow (Chap. 6) does not have this shape.

Although viscosity has a profound effect on fluid motion, the actual viscous stresses are quite small in magnitude even for oils, as shown in the following example.

EXAMPLE 1.8

Suppose that the fluid being sheared in Fig. 1.6 is SAE 30 oil at 20°C. Compute the shear stress in the oil if $V = 3$ m/s and $h = 2$ cm.

Solution

The shear stress is found from Eq. (1.23) by differentiating Eq. (1.26):

$$\tau = \mu\frac{du}{dy} = \frac{\mu V}{h} \tag{1}$$

From Table 1.4 for SAE 30 oil, $\mu = 0.29$ kg/(m · s). Then, for the given values of V and h, Eq. (1) predicts

$$\tau = \frac{[0.29 \text{ kg/(m} \cdot \text{s)}](3 \text{ m/s})}{0.02 \text{ m}} = 43 \text{ kg/(m} \cdot \text{s}^2)$$

$$= 43 \text{ N/m}^2 = 43 \text{ Pa} \qquad \textit{Ans.}$$

Although oil is very viscous, this is a modest shear stress, about 2400 times less than atmospheric pressure. Viscous stresses in gases and thin liquids are even smaller.

Variation of Viscosity with Temperature

Temperature has a strong effect and pressure a moderate effect on viscosity. The viscosity of gases and most liquids increases slowly with pressure. Water is anomalous in showing a very slight decrease below 30°C. Since the change in viscosity is only a few percent up to 100 atm, we shall neglect pressure effects in this book.

Gas viscosity increases with temperature. Two common approximations are the power law and the Sutherland law:

$$\frac{\mu}{\mu_0} \approx \begin{cases} \left(\dfrac{T}{T_0}\right)^n & \text{power law} \\[2ex] \dfrac{(T/T_0)^{3/2}(T_0 + S)}{T + S} & \text{Sutherland law} \end{cases} \tag{1.27}$$

where μ_0 is a known viscosity at a known absolute temperature T_0 (usually 273 K). The constants n and S are fit to the data, and both formulas are adequate over a wide range of temperatures. For air, $n \approx 0.7$ and $S \approx 110 \text{ K} = 199°\text{R}$. Other values are given in Ref. 3.

Liquid viscosity decreases with temperature and is roughly exponential, $\mu \approx ae^{-bT}$; but a better fit is the empirical result that $\ln \mu$ is quadratic in $1/T$, where T is absolute temperature

$$\ln \frac{\mu}{\mu_0} \approx a + b\left(\frac{T_0}{T}\right) + c\left(\frac{T_0}{T}\right)^2 \tag{1.28}$$

For water, with $T_0 = 273.16$ K, $\mu_0 = 0.001792$ kg/(m · s), suggested values are $a = -1.94$, $b = -4.80$, and $c = 6.74$, with accuracy about ±1 percent. The viscosity of water is tabulated in Table A.1. Curve-fit viscosity formulas for 355 organic liquids are given by Yaws et al. [34]. For further viscosity data, see Refs. 28 and 36.

Thermal Conductivity

Just as viscosity relates applied stress to resulting strain rate, there is a property called *thermal conductivity* k which relates the vector rate of heat flow per unit area $\mathbf{q}$ to the vector gradient of temperature ∇T. This proportionality, observed experimentally for fluids and solids, is known as *Fourier's law of heat conduction*

$$\mathbf{q} = -k\nabla T \tag{1.29a}$$

which can also be written as three scalar equations

$$q_x = -k\frac{\partial T}{\partial x} \qquad q_y = -k\frac{\partial T}{\partial y} \qquad q_z = -k\frac{\partial T}{\partial z} \tag{1.29b}$$

The minus sign satisfies the convention that heat flux is positive in the direction of decreasing temperature. Fourier's law is dimensionally consistent, and k has SI units of joules per second-meter-kelvin. Thermal conductivity k is a thermodynamic property and varies with temperature and pressure in much the same way as viscosity. The ratio k/k_0 can be correlated with T/T_0 in the same manner as Eqs. (1.27) and (1.28) for gases and liquids, respectively.

Further data on viscosity and thermal-conductivity variations can be found in Ref. 11.

Nonnewtonian Fluids

Fluids which do not follow the linear law of Eq. (1.23) are called *nonnewtonian* and are treated in books on rheology [6]. Figure 1.7*a* compares four examples with a newtonian fluid. A *dilatant,* or shear-thickening, fluid increases resistance with increasing applied stress. Alternately, a *pseudoplastic,* or shear-thinning, fluid decreases resistance with increasing stress. If the thinning effect is very strong, as with the dashed-line curve, the fluid is termed *plastic*. The limiting case of a plastic substance is one which requires a finite yield stress before it begins to flow. The linear-flow *Bingham plastic* idealization is shown, but the flow behavior after yield may also be nonlinear. An example of a yielding fluid is toothpaste, which will not flow out of the tube until a finite stress is applied by squeezing.

A further complication of nonnewtonian behavior is the transient effect shown in Fig. 1.7*b*. Some fluids require a gradually increasing shear stress to maintain a constant strain rate and are called *rheopectic*. The opposite case of a fluid which thins out with time and requires decreasing stress is termed *thixotropic*. We neglect nonnewtonian effects in this book; see Ref. 6 for further study.

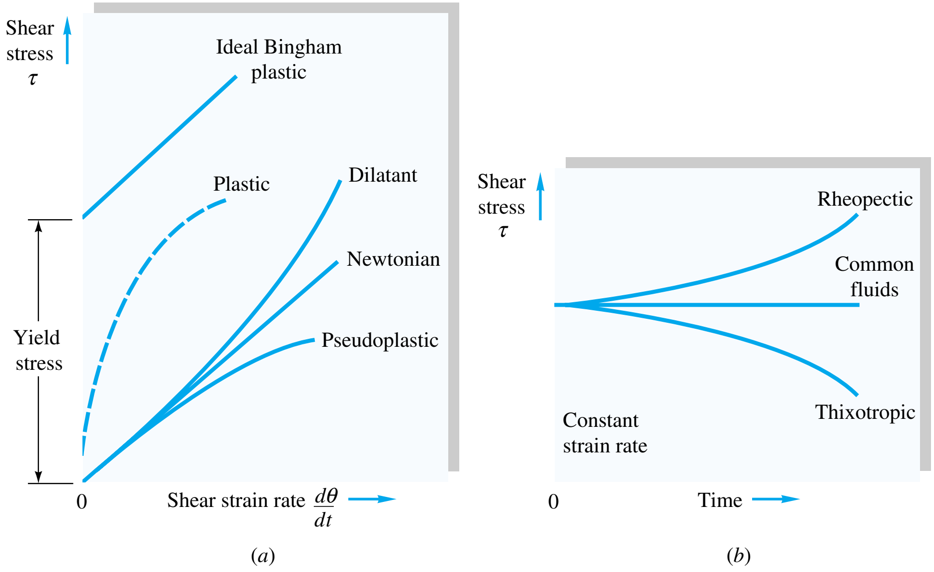


Fig. 1.7 Rheological behavior of various viscous materials: (*a*) stress versus strain rate; (*b*) effect of time on applied stress.

Surface Tension

A liquid, being unable to expand freely, will form an *interface* with a second liquid or gas. The physical chemistry of such interfacial surfaces is quite complex, and whole textbooks are devoted to this specialty [15]. Molecules deep within the liquid repel each other because of their close packing. Molecules at the surface are less dense and attract each other. Since half of their neighbors are missing, the mechanical effect is that the surface is in tension. We can account adequately for surface effects in fluid mechanics with the concept of surface tension.

If a cut of length dL is made in an interfacial surface, equal and opposite forces of magnitude $\Upsilon\, dL$ are exposed normal to the cut and parallel to the surface, where Υ is called the *coefficient of surface tension*. The dimensions of Υ are $\{F/L\}$, with SI units of newtons per meter and BG units of pounds-force per foot. An alternate concept is to open up the cut to an area dA; this requires work to be done of amount $\Upsilon\, dA$. Thus the coefficient Υ can also be regarded as the surface energy per unit area of the interface, in $\mathrm{N \cdot m/m^2}$ or $\mathrm{ft \cdot lbf/ft^2}$.

The two most common interfaces are water-air and mercury-air. For a clean surface at 20°C = 68°F, the measured surface tension is

$$\Upsilon = \begin{cases} 0.0050 \text{ lbf/ft} = 0.073 \text{ N/m} & \text{air-water} \\ 0.033 \text{ lbf/ft} = 0.48 \text{ N/m} & \text{air-mercury} \end{cases} \tag{1.30}$$

These are design values and can change considerably if the surface contains contaminants like detergents or slicks. Generally Υ decreases with liquid temperature and is zero at the critical point. Values of Υ for water are given in Fig. 1.8.

If the interface is curved, a mechanical balance shows that there is a pressure difference across the interface, the pressure being higher on the concave side, as illustrated in Fig. 1.9. In Fig. 1.9*a,* the pressure increase in the interior of a liquid cylinder is balanced by two surface-tension forces

$$2RL\,\Delta p = 2\Upsilon L$$

or

$$\Delta p = \frac{\Upsilon}{R} \tag{1.31}$$

We are not considering the weight of the liquid in this calculation. In Fig. 1.9*b,* the pressure increase in the interior of a spherical droplet balances a ring of surface-tension force

$$\pi R^2\,\Delta p = 2\pi R\Upsilon$$

or

$$\Delta p = \frac{2\Upsilon}{R} \tag{1.32}$$

We can use this result to predict the pressure increase inside a soap bubble, which has *two* interfaces with air, an inner and outer surface of nearly the same radius R:

$$\Delta p_{\text{bubble}} \approx 2\,\Delta p_{\text{droplet}} = \frac{4\Upsilon}{R} \tag{1.33}$$

Figure 1.9*c* shows the general case of an arbitrarily curved interface whose principal radii of curvature are R_1 and R_2. A force balance normal to the surface will show that the pressure increase on the concave side is

$$\Delta p = \Upsilon({R_1}^{-1} + {R_2}^{-1}) \tag{1.34}$$

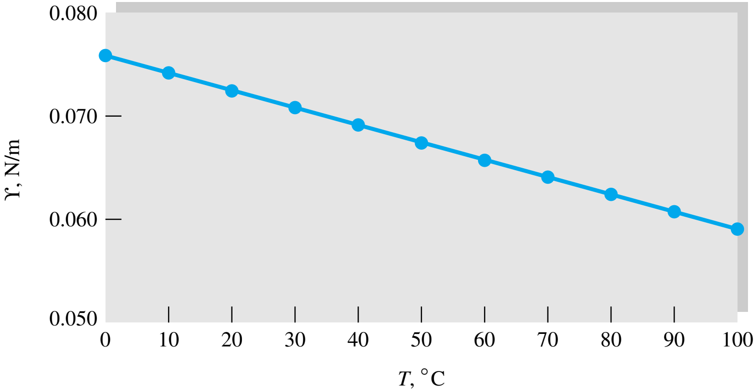


Fig. 1.8 Surface tension of a clean air-water interface. Data from Table A.5.

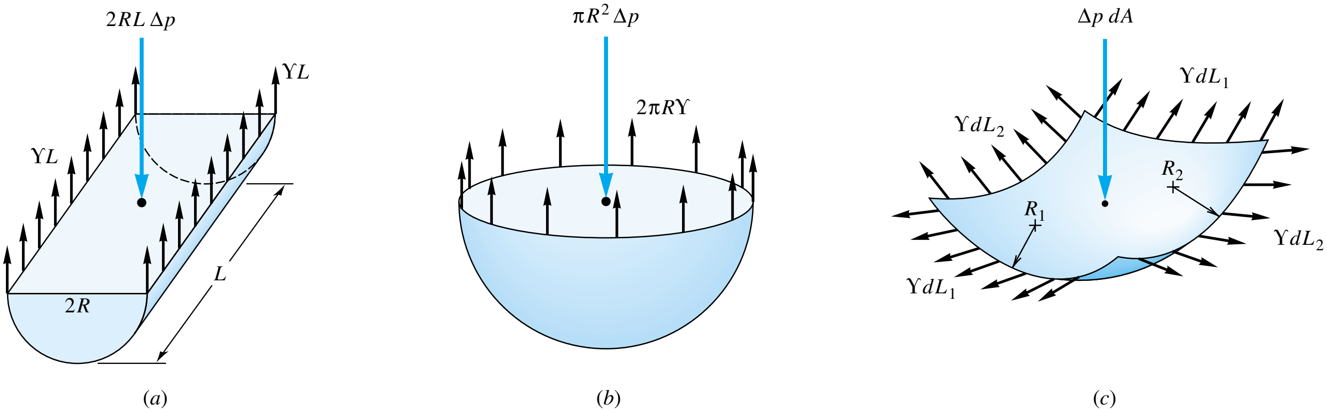


Fig. 1.9 Pressure change across a curved interface due to surface tension: (*a*) interior of a liquid cylinder; (*b*) interior of a spherical droplet; (*c*) general curved interface.

Equations (1.31) to (1.33) can all be derived from this general relation; e.g., in Eq. (1.31), $R_1 = R$ and $R_2 = \infty$.

A second important surface effect is the *contact angle* θ which appears when a liquid interface intersects with a solid surface, as in Fig. 1.10. The force balance would then involve both Υ and θ. If the contact angle is less than 90°, the liquid is said to *wet* the solid; if $\theta > 90°$, the liquid is termed *nonwetting*. For example, water wets soap but does not wet wax. Water is extremely wetting to a clean glass surface, with $\theta \approx 0°$. Like Υ, the contact angle θ is sensitive to the actual physicochemical conditions of the solid-liquid interface. For a clean mercury-air-glass interface, $\theta = 130°$.

Example 1.9 illustrates how surface tension causes a fluid interface to rise or fall in a capillary tube.

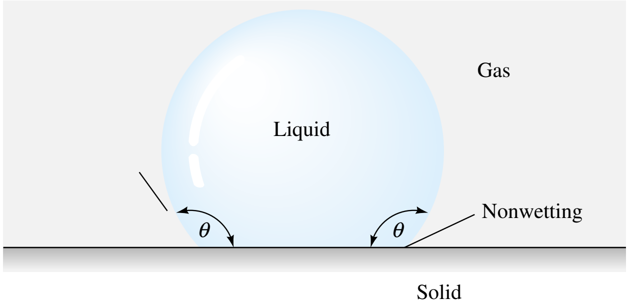


Fig. 1.10 Contact-angle effects at liquid-gas-solid interface. If $\theta < 90°$, the liquid "wets" the solid; if $\theta > 90°$, the liquid is nonwetting.

EXAMPLE 1.9

Derive an expression for the change in height h in a circular tube of a liquid with surface tension Υ and contact angle θ, as in Fig. E1.9.

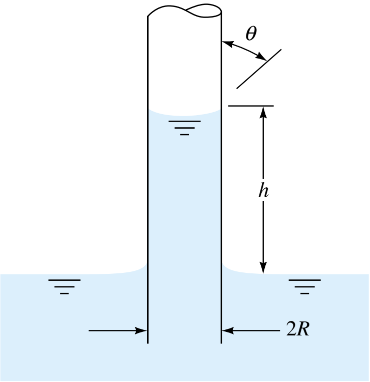


Fig. E1.9

Solution

The vertical component of the ring surface-tension force at the interface in the tube must balance the weight of the column of fluid of height h

$$2\pi R\Upsilon \cos\theta = \gamma\pi R^2 h$$

Solving for h, we have the desired result

$$h = \frac{2\Upsilon \cos\theta}{\gamma R} \qquad \textit{Ans.}$$

Thus the capillary height increases inversely with tube radius R and is positive if $\theta < 90°$ (wetting liquid) and negative (capillary depression) if $\theta > 90°$.

Suppose that $R = 1$ mm. Then the capillary rise for a water-air-glass interface, $\theta \approx 0°$, $\Upsilon = 0.073$ N/m, and $\rho = 1000$ kg/m^3 is

$$h = \frac{2(0.073\ \text{N/m})(\cos 0°)}{(1000\ \text{kg/m}^3)(9.81\ \text{m/s}^2)(0.001\ \text{m})} = 0.015\ (\text{N}\cdot\text{s}^2)/\text{kg} = 0.015\ \text{m} = 1.5\ \text{cm}$$

For a mercury-air-glass interface, with $\theta = 130°$, $\Upsilon = 0.48$ N/m, and $\rho = 13{,}600$ kg/m^3, the capillary rise is

$$h = \frac{2(0.48)(\cos 130°)}{13{,}600(9.81)(0.001)} = -0.46\ \text{cm}$$

When a small-diameter tube is used to make pressure measurements (Chap. 2), these capillary effects must be corrected for.

Vapor Pressure

Vapor pressure is the pressure at which a liquid boils and is in equilibrium with its own vapor. For example, the vapor pressure of water at 68°F is 49 lbf/ft^2, while that of mercury is only 0.0035 lbf/ft^2. If the liquid pressure is greater than the vapor pressure, the only exchange between liquid and vapor is evaporation at the interface. If, however, the liquid pressure falls below the vapor pressure, vapor bubbles begin to appear in the liquid. If water is heated to 212°F, its vapor pressure rises to 2116 $\mathrm{lbf/ft^2}$, and thus water at normal atmospheric pressure will boil. When the liquid pressure is dropped below the vapor pressure due to a flow phenomenon, we call the process *cavitation*. As we shall see in Chap. 2, if water is accelerated from rest to about 50 ft/s, its pressure drops by about 15 $\mathrm{lbf/in^2}$, or 1 atm. This can cause cavitation.

The dimensionless parameter describing flow-induced boiling is the *cavitation number*

$$\mathrm{Ca} = \frac{p_a - p_v}{\frac{1}{2}\rho V^2} \tag{1.35}$$

where p_a = ambient pressure
p_v = vapor pressure
V = characteristic flow velocity

Depending upon the geometry, a given flow has a critical value of Ca below which the flow will begin to cavitate. Values of surface tension and vapor pressure of water are given in Table A.5. The vapor pressure of water is plotted in Fig. 1.11.

Figure 1.12*a* shows cavitation bubbles being formed on the low-pressure surfaces of a marine propeller. When these bubbles move into a higher-pressure region, they collapse implosively. Cavitation collapse can rapidly spall and erode metallic surfaces and eventually destroy them, as shown in Fig. 1.12*b*.

No-Slip and No-Temperature-Jump Conditions

When a fluid flow is bounded by a solid surface, molecular interactions cause the fluid in contact with the surface to seek momentum and energy equilibrium with that surface. All liquids essentially are in equilibrium with the surface they contact. All gases are, too,

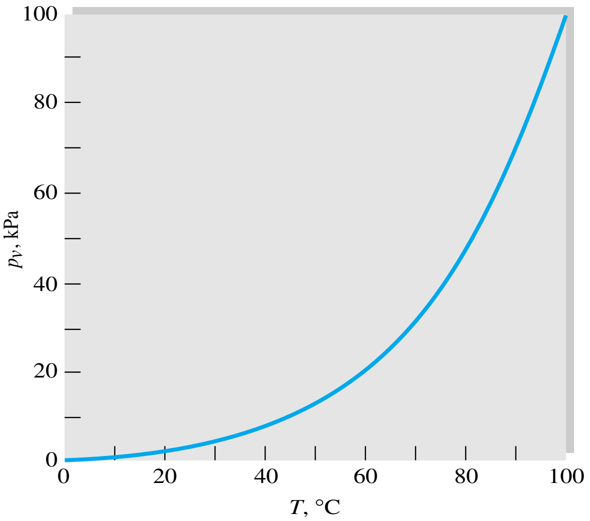


Fig. 1.11 Vapor pressure of water. Data from Table A.5.

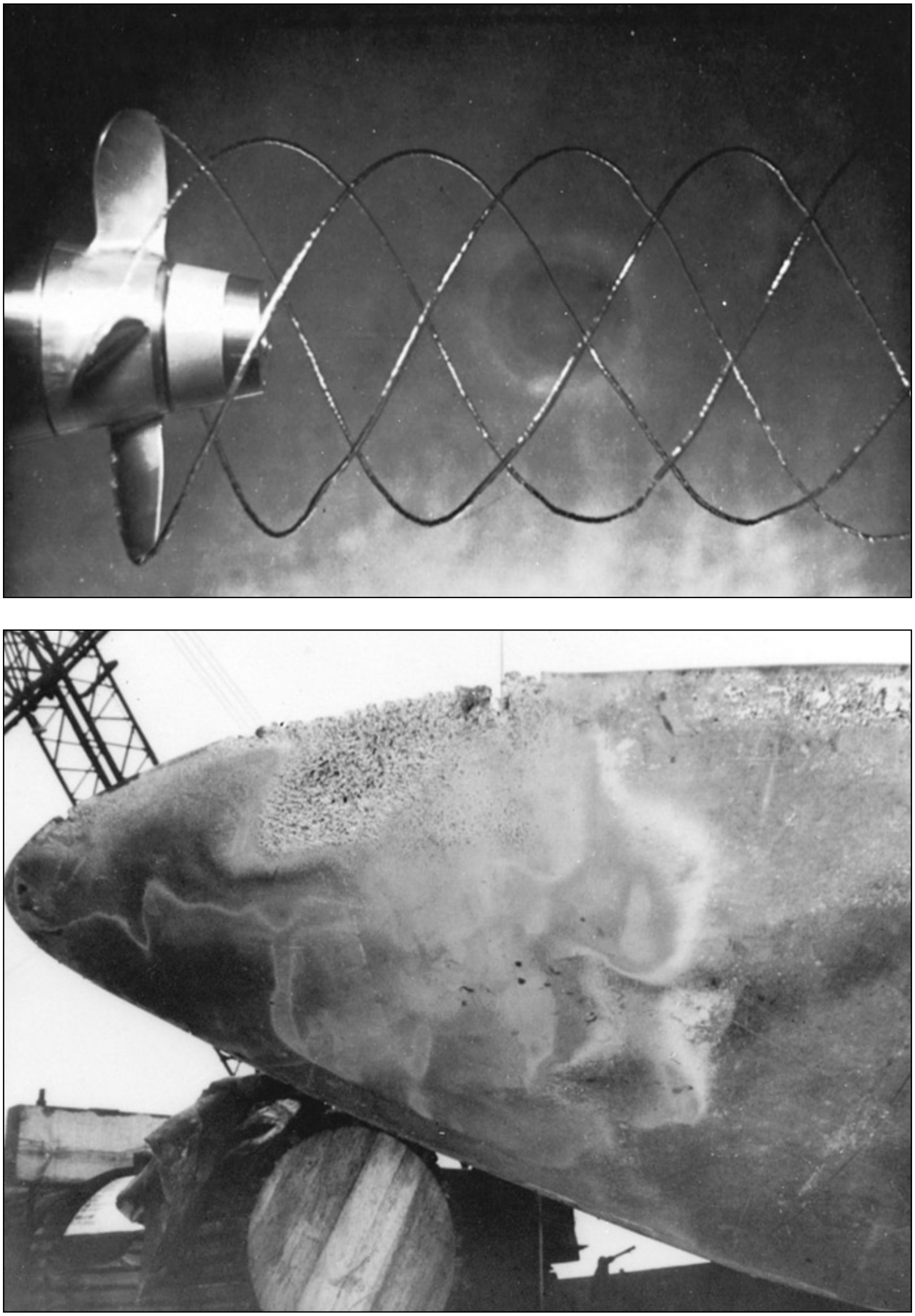


Fig. 1.12 Two aspects of cavitation bubble formation in liquid flows: (*a*) Beauty: spiral bubble sheets form from the surface of a marine propeller. *(Courtesy of the Garfield Thomas Water Tunnel, Pennsylvania State University);* (*b*) ugliness: collapsing bubbles erode a propeller surface. *(Courtesy of Thomas T. Huang, David Taylor Research Center.)*

except under the most rarefied conditions [8]. Excluding rarefied gases, then, all fluids at a point of contact with a solid take on the velocity and temperature of that surface

$$\mathbf{V}_{\text{fluid}} \equiv \mathbf{V}_{\text{wall}} \qquad T_{\text{fluid}} \equiv T_{\text{wall}} \tag{1.36}$$

These are called the *no-slip* and *no-temperature-jump conditions,* respectively. They serve as *boundary conditions* for analysis of fluid flow past a solid surface (Chap. 6). Figure 1.13 illustrates the no-slip condition for water flow past the top and bottom surfaces of a fixed thin plate. The flow past the upper surface is disorderly, or turbulent, while the lower surface flow is smooth, or laminar.[7] In both cases there is clearly no slip at the wall, where the water takes on the zero velocity of the fixed plate. The velocity profile is made visible by the discharge of a line of hydrogen bubbles from the wire shown stretched across the flow.

To decrease the mathematical difficulty, the no-slip condition is partially relaxed in the analysis of inviscid flow (Chap. 8). The flow is allowed to "slip" past the surface but not to permeate through the surface

$$V_{\text{normal}}(\text{fluid}) \equiv V_{\text{normal}}(\text{solid}) \tag{1.37}$$

while the tangential velocity V_t is allowed to be independent of the wall. The analysis is much simpler, but the flow patterns are highly idealized.

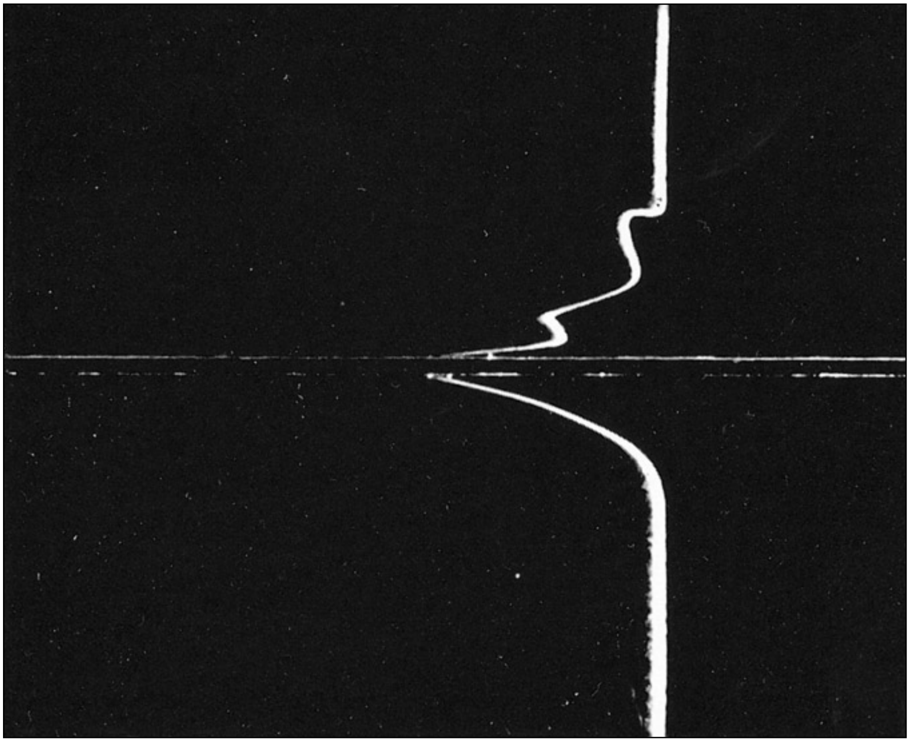


Fig. 1.13 The no-slip condition in water flow past a thin fixed plate. The upper flow is turbulent; the lower flow is laminar. The velocity profile is made visible by a line of hydrogen bubbles discharged from the wire across the flow. *[From* Illustrated Experiments in Fluid Mechanics *(The NCFMF Book of Film Notes), National Committee for Fluid Mechanics Films, Education Development Center, Inc., copyright 1972.]*

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[7] Laminar and turbulent flows are studied in Chaps. 6 and 7.

Speed of Sound

In gas flow, one must be aware of *compressibility* effects (significant density changes caused by the flow). We shall see in Sec. 4.2 and in Chap. 9 that compressibility becomes important when the flow velocity reaches a significant fraction of the speed of sound of the fluid. The *speed of sound a* of a fluid is the rate of propagation of small-disturbance pressure pulses ("sound waves") through the fluid. In Chap. 9 we shall show, from momentum and thermodynamic arguments, that the speed of sound is defined by

$$a^2 = \left(\frac{\partial p}{\partial \rho}\right)_s = k\left(\frac{\partial p}{\partial \rho}\right)_T \qquad k = \frac{c_p}{c_v} \tag{1.38}$$

This is true for either a liquid or a gas, but it is for *gases* that the problem of compressibility occurs. For an ideal gas, Eq. (1.10), we obtain the simple formula

$$a_{\text{ideal gas}} = (kRT)^{1/2} \tag{1.39}$$

where R is the gas constant, Eq. (1.11), and T the absolute temperature. For example, for air at 20°C, $a = \{(1.40)[287\ \text{m}^2/(\text{s}^2 \cdot \text{K})](293\ \text{K})\}^{1/2} \approx 343$ m/s (1126 ft/s = 768 mi/h). If, in this case, the air velocity reaches a significant fraction of a, say, 100 m/s, then we must account for compressibility effects (Chap. 9). Another way to state this is to account for compressibility when the *Mach number* $\text{Ma} = V/a$ of the flow reaches about 0.3.

The speed of sound of water is tabulated in Table A.5. The speed of sound of air (or any approximately perfect gas) is simply calculated from Eq. (1.39).

1.8 Basic Flow-Analysis Techniques

There are three basic ways to attack a fluid-flow problem. They are equally important for a student learning the subject, and this book tries to give adequate coverage to each method:

1. Control-volume, or *integral* analysis (Chap. 3)
2. Infinitesimal system, or *differential* analysis (Chap. 4)
3. Experimental study, or *dimensional* analysis (Chap. 5)

In all cases, the flow must satisfy the three basic laws of mechanics[8] plus a thermodynamic state relation and associated boundary conditions:

1. Conservation of mass (continuity)
2. Linear momentum (Newton's second law)
3. First law of thermodynamics (conservation of energy)
4. A state relation like $\rho = \rho(p, T)$
5. Appropriate boundary conditions at solid surfaces, interfaces, inlets, and exits

In integral and differential analyses, these five relations are modeled mathematically and solved by computational methods. In an experimental study, the fluid itself performs this task without the use of any mathematics. In other words, these laws are believed to be fundamental to physics, and no fluid flow is known to violate them.

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[8] In fluids which are variable mixtures of components, such as seawater, a fourth basic law is required, *conservation of species*. For an example of salt conservation analysis, see Chap. 4, Ref. 16.